Proof Mini-Unit

Indirect Proof

OBJECTIVE

- To write indirect proofs

VOCABULARY

Indirect reasoning – all possibilities are considered and then all but one are proved false. The remaining possibility must be true.

Indirect proof – a type of proof that uses indirect reasoning (Use this when the statement and its opposite are the only possibilities.)

KEY CONCEPT

Steps for indirect proof:

1. Identify the conjecture to be proven.
2. Assume the opposite (negation) of the conclusion is true.
3. Use direct reasoning to show that the assumption leads to a contradiction.
4. Conclude that since the assumption is false, the original conjecture must be true.

CLASS WORK

Complete the second step of an indirect proof: temporarily assume the negation of the prove statements.

1. There are fewer than 11 pencils in the box.
   Temporarily assume that there are at least 11 pencils in the box.

2. ΔRST is not an isosceles triangle.
   Temporarily assume that ΔRST is an isosceles triangle.

CLASS WORK

Identify the two statements that contradict each other.

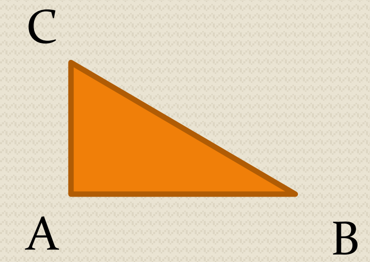
3. I. $m\angle B \leq 90$.
   II. $\angle B$ is acute.
   III. $\angle B$ is a right angle.

II and III contradict each other.

4. I. The orthocenter for ΔABC is outside the triangle.
   II. The centroid for ΔABC is inside the triangle.
   III. ΔABC is an acute triangle.

I and III contradict each other.

CLASS WORK



Write an indirect proof that a triangle cannot have two right angles.

Given: Triangle ABC.

Prove: A triangle cannot have two right angles.

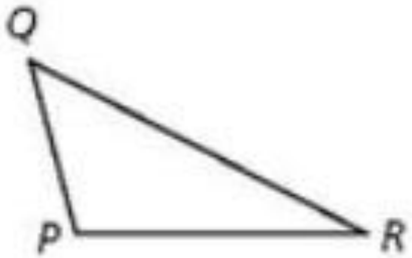
Step 2: Temporarily assume that a triangle can have two right angles.

Step 3: $m\angle A + m\angle B + m\angle C = 180$ by the Triangle angle sum theorem.
By substitution, $90 + m\angle B + 90 = 180$.
By CLT, $m\angle B + 180 = 180$.
By the subtraction property of equality, $m\angle B = 0$.

Step 4: Since this contradicts the protractor postulate, the temporary assumption is false and it must be true that a triangle cannot have two right angles.

KEY CONCEPT

Theorem 5-5-2



Given: $m\angle P > m\angle R$
Prove: $QR > QP$

Indirect Proof:
Assume $QR \ngtr QP$. This means that either $QR < QP$ or $QR = QP$.

Case 1 If $QR < QP$, then $m\angle P < m\angle R$ because the larger angle is opposite the longer side. This contradicts the given information. So $QR \nless QP$.

Case 2 If $QR = QP$, then $m\angle P = m\angle R$ by the Isosceles Triangle Theorem. This also contradicts the given information, so $QR \neq QP$.

The assumption $QR \ngtr QP$ is false. Therefore $QR > QP$.

Class Work

In each set of statements, name the two that contradict each other.

36. $\triangle PQR$ is a right triangle.
$\triangle PQR$ is a scalene triangle.
$\triangle PQR$ is an acute triangle.

37. $\angle Y$ is supplementary to $\angle Z$.
$m\angle Y < 90^\circ$
$\angle Y$ is an obtuse angle.

38. $\triangle JKL$ is isosceles with base $\overline{JL}$.
In $\triangle JKL$, $m\angle K > m\angle J$
In $\triangle JKL$, $JK > LK$

39. $\overline{AB} \perp \overline{BC}$
$\overline{AB} \cong \overline{CD}$
$\overline{AB} \parallel \overline{BC}$

40. Figure A is a polygon.
Figure A is a triangle.
Figure A is a quadrilateral.

41. x is even.
x is a multiple of 4.
x is prime.

SUMMARY

- In an indirect proof, you first assume temporarily the opposite of what you want to prove. Then you show that this temporary assumption leads to a contradiction, so the prove statement must be true.