SECTION 1 - 6

Midpoint and Distance Formulas

OBJECTIVES

» 1. To find the midpoint of a segment
2. To find the distance between two points on the coordinate plane
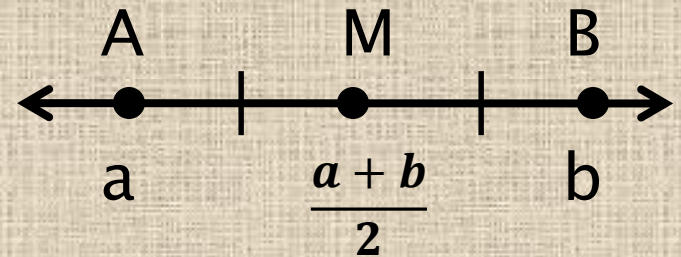
KEY CONCEPT

Midpoint formula on a number line – average or mean of the endpoints

Formula: $\dfrac{a+b}{2}$

Given: a = −7; b = 4

$$\frac{-7+4}{2} = \frac{-3}{2} = -1.5$$

A   M   B
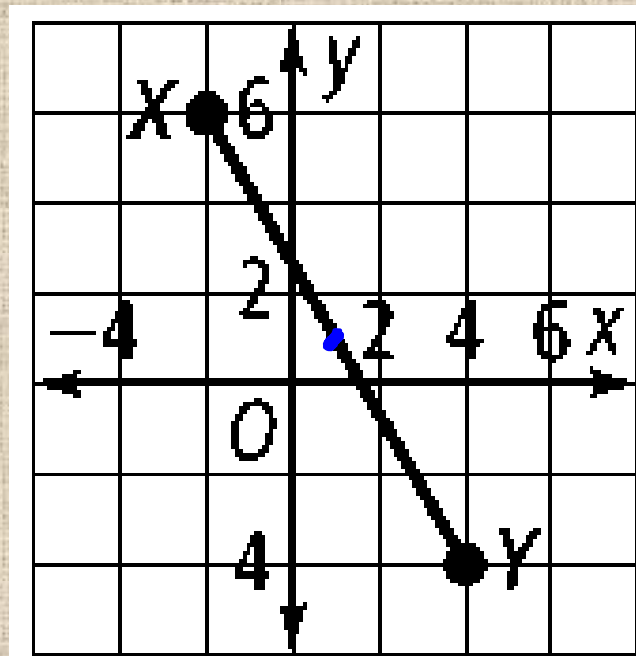
a $\dfrac{a+b}{2}$ b

KEY CONCEPT

Midpoint formula in the coordinate plane – average of the x–coordinates and the average of the y–coordinates of the endpoints

Formula:

$$\left[\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right]$$

Given: X(–2,6);Y(4,–4)

$$\left(\frac{-2+4}{2}, \frac{6-4}{2} \right) = (1,1)$$

CLASS WORK

1. Find the coordinates of the midpoint of $\overline{AB}$ if A(2.8,1.1) and B(−3.4,5.7)

2. The coordinates of point Y are (−10,5).The midpoint of $\overline{XY}$ is (3, −5). Find the coordinates of point X.

   (−10, 5)   (3, −5)

CLASS WORK

1. Find the coordinates of the midpoint of $\overline{AB}$ if
A(2.8,1.1) and B(−3.4,5.7)

$$\left(\frac{2.8 + {}^-3.4}{2}, \frac{1.1 + 5.7}{2}\right) \rightarrow \left(\frac{-0.6}{2}, \frac{6.8}{2}\right) \rightarrow \left(-0.3, 3.4\right)$$

2. The coordinates of point Y are (−10,5).The midpoint of $\overline{XY}$ is (3, −5). Find the coordinates of point X.

Y(−10, 5) M(3, −5) X(16, −15)
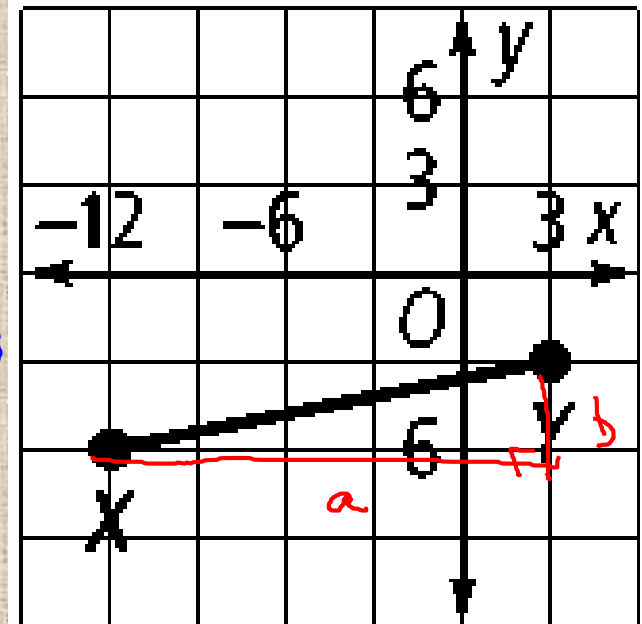
KEY CONCEPT

Distance formula– distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given: X(–12,–6);Y(3,–3)

$d = \sqrt{(3 + 12)^2 + (-3 + 6)^2}$

$d = \sqrt{225 + 9} = \sqrt{234}$

$d = 3\sqrt{26} \approx 15.3$ units

CLASS WORK

3. In the previous graph, each unit represents 10 miles. How would you find the distance in miles?

4. Find the distance between the following pair of points: J(−5,5) and K(−3,−2)

5. Two buses leave the station at the same time. One travels 5 miles east and then 2 miles south and the other travels 9 miles west and 4 miles north. What is the distance between the two buses?

CLASS WORK

3. In the previous graph, each unit represents 10 miles. How would you find the distance in miles?

$$15.3 \text{ un} \times \frac{10 mi}{1 un} = \boxed{153 \text{ miles}}$$

4. Find the distance between the following pair of points: J(–5,5) and K(–3,–2)

$$d = \sqrt{(-3+5)^2 + (-2-5)^2} = \sqrt{4+49} = \boxed{\sqrt{53} \approx 7.3 \text{ units}}$$

5. Two buses leave the station at the same time. One travels 5 miles east and then 2 miles south and the other travels 9 miles west and 4 miles north. What is the distance between the two buses?
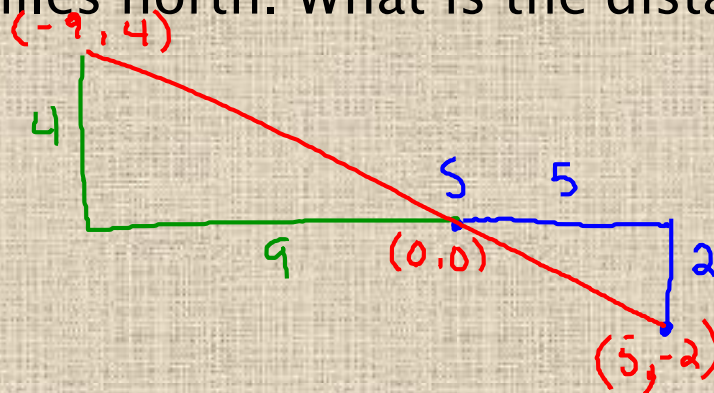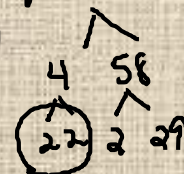
$$d = \sqrt{(5+9)^2 + (-2-4)^2}$$
$$d = \sqrt{196 + 36} = \sqrt{232}$$
$$\boxed{d = 2\sqrt{58} \approx 15.2 mi}$$

(–9, 4)

4

9

5   5

(0,0)

2

(5,–2)

4   58

22   2   29

SUMMARY

1. Midpoint on a number line: $\frac{a+b}{2}$

2. Midpoint in coordinate plane: $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

3. Distance formula: $d=\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

EXIT PROBLEM

Given: A(20, −4) and B(−4,3)

$$x_1 \quad y_1 \qquad x_2 \quad y_2$$

a. Find the midpoint of $\overline{AB}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \rightarrow \left(\frac{20 - 4}{2}, \frac{-4 + 3}{2} \right) \rightarrow \left(8, -\frac{1}{2} \right)$$

b. Find the distance between points A and B.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-4 - 20)^2 + (3 + 4)^2}$$

$$d = \sqrt{576 + 49} = \sqrt{625} = 25 \text{ units}$$

ANSWER SLIDE

1. (–0.3, 3.4)
2. (16, –15)
3. Approx. 153 miles
4. $\sqrt{53} \approx 7.3$ units
5. $2\sqrt{58} \approx 15.23$ miles

Exit: midpoint: $(8, -\frac{1}{2})$ AB = 25 units

LEARNING RUBRIC

- Got It: Represents and /or applies formulas for comparison purposes, and in complex real-world situations
- Almost There: Given and endpoint and the midpoint, find the second endpoint; finds the distance between two given points with no graph
- Moving Forward: Finds midpoint between two given points; finds the distance between two points on a graph
- Getting Started: Finds the midpoint on a graphed segment

HOMEWORK

- Page 47 – 49
- 12, 14, 16, 22, 24, 26, 30, 34, 36, 40