Proof Mini-unit

Introduction to Coordinate Proof

OBJECTIVES

1. To use formulas to classify polygons on the coordinate plane
2. To prove geometric concepts using coordinate proofs

KEY CONCEPT

take note

Key Concept Formulas and the Coordinate Plane

Formula	When to Use It
Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	To determine whether • sides are congruent • diagonals are congruent
Midpoint Formula $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	To determine • the coordinates of the midpoint of a side • whether diagonals bisect each other
Slope Formula $m = \frac{y_2 - y_1}{x_2 - x_1}$	To determine whether • opposite sides are parallel • diagonals are perpendicular • sides are perpendicular

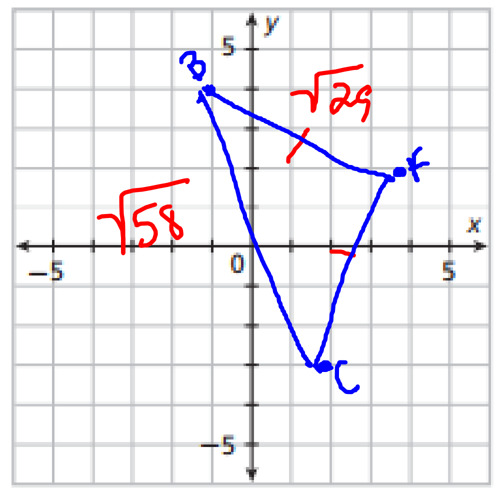
EXAMPLE

G-GPE.2.4

1 EXAMPLE Proving or Disproving a Statement

Prove or disprove that the triangle with vertices $A(4, 2)$, $B(-1, 4)$, and $C(2, -3)$ is an isosceles triangle.

A Plot the vertices and draw the triangle.



B Use the distance formula to find the length of each side of $\triangle ABC$.

$$AB = \sqrt{(-1-4)^2 + (4-2)^2} = \sqrt{(-5)^2 + 2^2} = \sqrt{29}$$

$$BC = \sqrt{(2+1)^2 + (-3-4)^2} = \sqrt{9+49} = \sqrt{58}$$

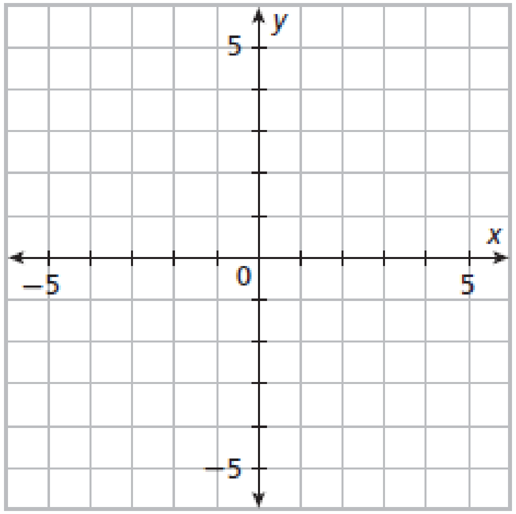
$$AC = \sqrt{(2-4)^2 + (-3-2)^2} = \sqrt{4+25} = \sqrt{29}$$

C Draw a conclusion based on your results. State whether or not the triangle is isosceles and why.

$\triangle ABC$ is isosceles because $\overline{AB} \cong \overline{AC}$

PRACTICE

1. Prove or disprove that the triangle with vertices $R(-2, -2)$, $S(1, 4)$, and $T(4, -5)$ is an equilateral triangle.



2. Refer to the triangle you drew in Exercise 1 to prove or disprove that the triangle with vertices $R(-2, -2)$, $S(1, 4)$, and $T(4, -5)$ is a right triangle.

PRACTICE

1. Prove or disprove that the triangle with vertices $R(-2, -2)$, $S(1, 4)$, and $T(4, -5)$ is an equilateral triangle.

A) Graph given Points

B) Use distance formula to find side lengths

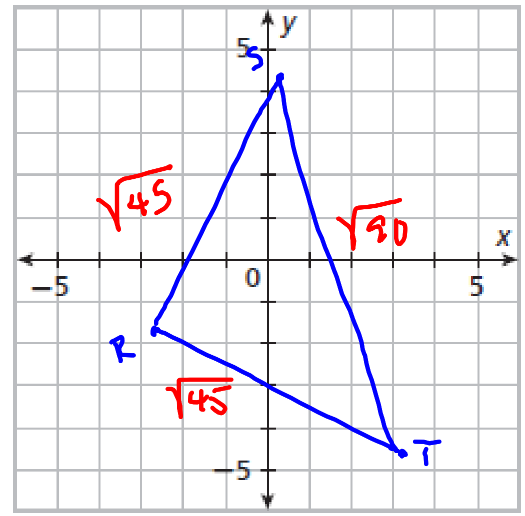
$RS = \sqrt{(1+2)^2 + (4+2)^2} = \sqrt{9+36} = \sqrt{45}$

$ST = \sqrt{(4-1)^2 + (-5-4)^2} = \sqrt{9+81} = \sqrt{90}$

$RT = \sqrt{(4+2)^2 + (-5+2)^2} = \sqrt{36+9} = \sqrt{45}$

C) Conclusion:

$\triangle RST$ is not equilateral because $RS = RT \neq ST$.



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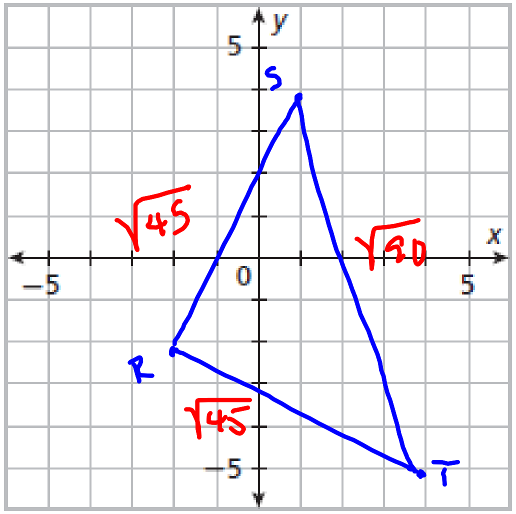

A) Graph given Points

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$RS = \sqrt{(1+2)^2 + (4+2)^2} = \sqrt{9+36} = \sqrt{45}$

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c) Conclusion:

$\triangle RST$ is not equilateral because $RS = RT \neq ST$.

2. Refer to the triangle you drew in Exercise 1 to prove or disprove that the triangle with vertices $R(-2, -2)$, $S(1, 4)$, and $T(4, -5)$ is a right triangle.

B) Find slopes of each side

$\overline{RS}: m = \frac{4+2}{1+2} = \frac{6}{3} = \boxed{2}$

$\overline{ST}: m = \frac{-5-4}{4-1} = \frac{-9}{3} = -3$

$\overline{RT}: m = \frac{-5+2}{4+2} = \frac{-3}{6} = \boxed{-\frac{1}{2}}$

c) Conclusion: $\triangle RST$ is a right $\triangle$ because $\overline{RS} \perp \overline{RT}$, so $\angle R$ is a right $\angle$

KEY CONCEPT

In order to use coordinate proof for Theorems, we must use variables when naming the coordinates of a figure to show that relationships are true for a general case.

Strategies for Positioning Figures in the Coordinate Plane

- Use the origin as a vertex, keeping the figure in Quadrant I.
- Center the figure at the origin.
- Center a side of the figure at the origin.
- Use one or both axes as sides of the figure.

KEY CONCEPT

M is the midpoint of $\overline{QP}$. N is the midpoint of $\overline{QR}$. Find *MN*.

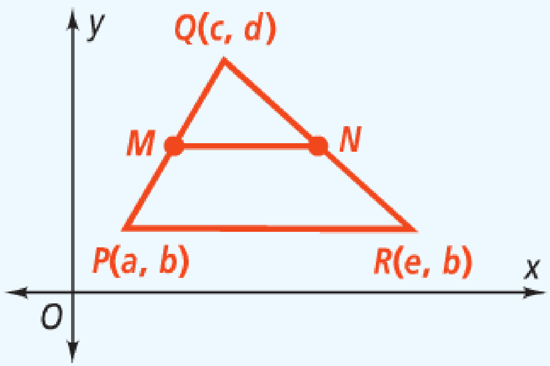


Figure 1

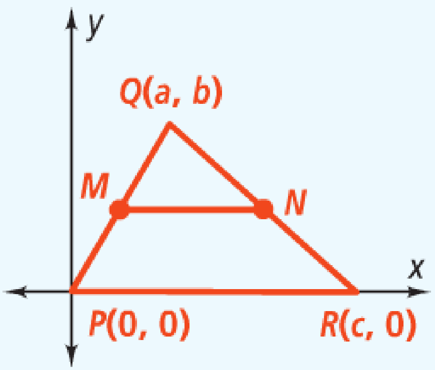


Figure 2

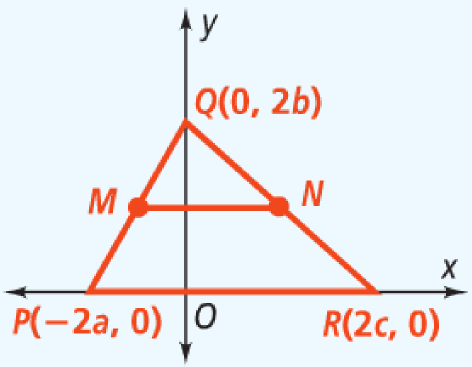


Figure 3

Figure 1 does not use the axes, so it requires more variables. Figures 2 and 3 have good placement. In Figure 2, the midpoint coordinates are $M\left(\frac{a}{2}, \frac{b}{2}\right)$ and $N\left(\frac{a+c}{2}, \frac{b}{2}\right)$. In Figure 3, the coordinates are $M(-a, b)$ and $N(c, b)$. You can see that Figure 3 is the easiest to work with.

EXAMPLE

G-GPE.2.4

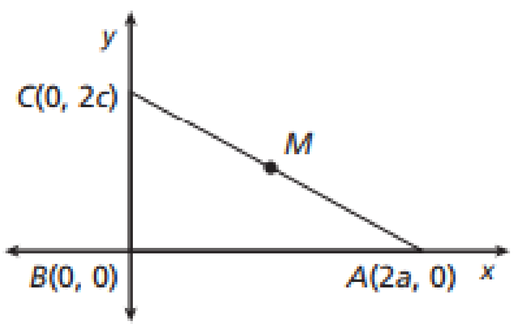
2 EXAMPLE Writing a Coordinate Proof

Prove that in a right triangle, the midpoint of the hypotenuse is equidistant from all three vertices.

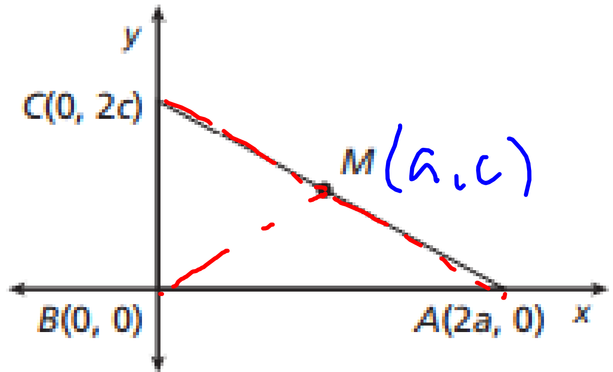
A Assign coordinates to the figure.

Let the triangle be $\triangle ABC$. Since the triangle is a right triangle, assume $\angle B$ is a right angle. Place $\angle B$ at the origin and place the legs along the positive x- and y-axes.

Since the proof involves a midpoint, use multiples of 2 in assigning coordinates to A and C, as shown.



EXAMPLE



B Let M be the midpoint of the hypotenuse, $\overline{AC}$. Use the midpoint formula to find the coordinates of M.

$$M\left(\frac{0 + 2a}{2}, \frac{2c + 0}{2}\right) = M(a, c)$$

C Use the distance formula to find MA, MB, and MC.

$$MA = \sqrt{(2a - a)^2 + (0 - c)^2} = \sqrt{a^2 + c^2}$$

$$MB = \sqrt{(a - 0)^2 + (c - 0)^2} = \sqrt{a^2 + c^2}$$

$$MC = \sqrt{(a - 0)^2 + (c - 2c)^2} = \sqrt{a^2 + c^2}$$

So, the midpoint of the hypotenuse is equidistant from all three vertices because

$MA = MB = MC$

CHALLENGE

14. Coordinate Geometry. The coordinates of the vertices of a triangle are $K(2, 3)$, $L(-2, -1)$, and $M(5, 1)$.

a. Find the coordinates of N, the midpoint of KM, and P, the midpoint of LM.

b. Show that $\overline{NP} \parallel \overline{KL}$.

c. Show that $NP = \frac{1}{2} KL$.

SUMMARY

1. Use the distance, midpoint and slope formulas to classify polygons in the coordinate plane.
2. Use variables when naming the coordinates of a figure in order to show that relationships are true for a general case.