

9-7

Dilations

# OBJECTIVES

- » To identify and draw dilations
- To dilate figures on the coordinate plane

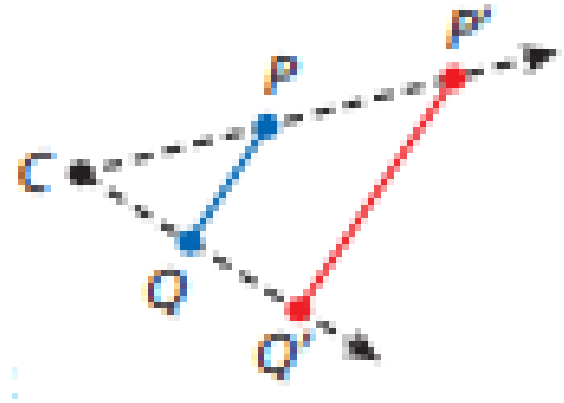
# DILATION

**Dilation (Similarity transformation)** – transformation in which the lines connecting every point with its image point all intersect at a point called the **center of dilation**. The **scale factor (k)** of a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage. This is not a rigid motion.

An **enlargement** has a scale factor greater than 1.

A **reduction (contraction)** has a scale factor greater than 0 but less than 1.

The rule in either case is  $(x, y) \rightarrow (kx, ky)$

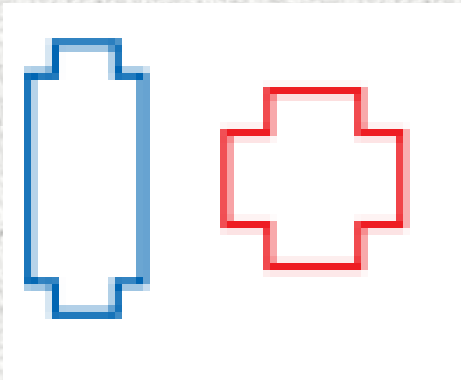


$$\text{Scale factor} = k = \frac{P'Q'}{PQ}$$

# EXAMPLES

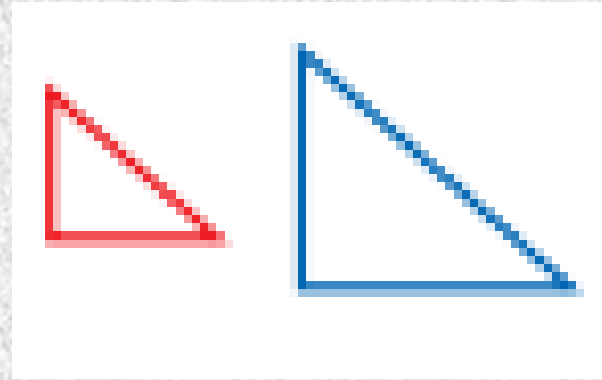
Tell whether each transformation appears to be a dilation. Explain.

1.



No. The sides do not change by the same scale factor.

2.



Yes. Every side of the image appears to be increased by the same scale factor.

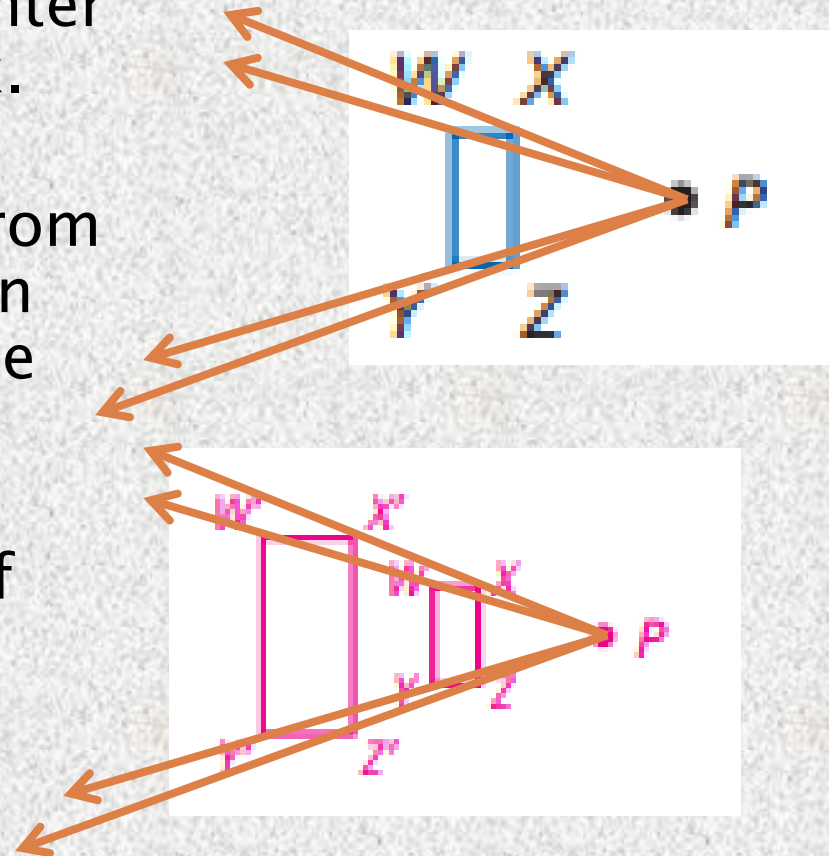
# EXAMPLES

To draw a dilation with a scale factor of 2:

Step 1. Draw a ray from the center of dilation through each vertex.

Step 2. Measure the distance from each vertex to center of dilation and mark off twice this distance on corresponding ray.

Step 3. Connect the vertices of the image.

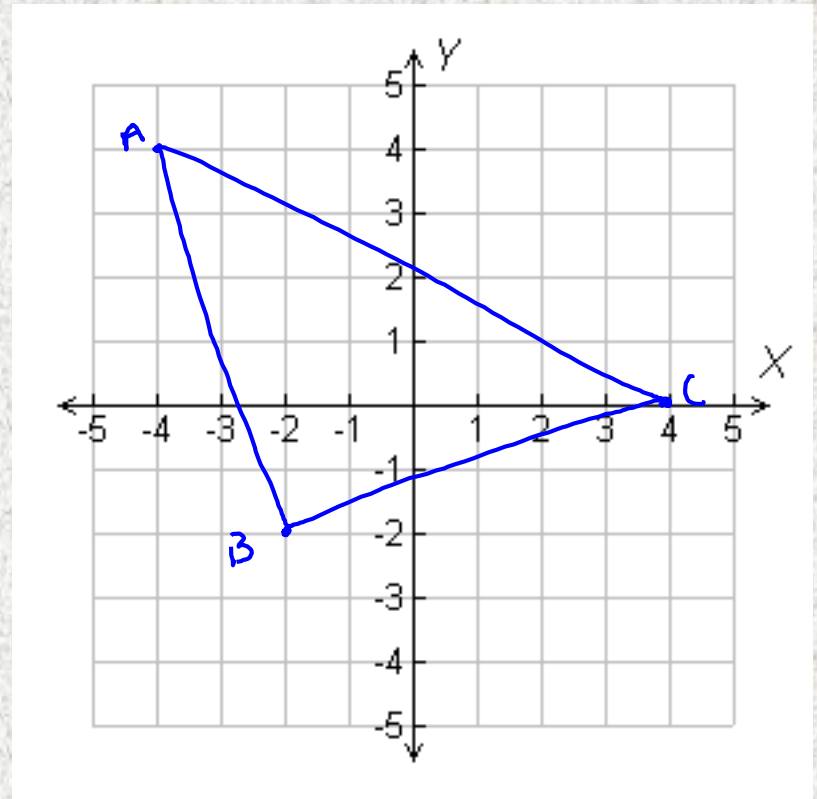


# EXAMPLES

3. Draw the image of the triangle with vertices  $A(-4, 4)$ ;  $B(-2, -2)$ ;  $C(4, 0)$  under a dilation with a scale factor of  $\frac{1}{2}$  centered at the origin.

The rule is:

$$(x, y) \rightarrow (kx, ky)$$



# EXAMPLES

3. Draw the image of the triangle with vertices  $A(-4, 4)$ ;  $B(-2, -2)$ ;  $C(4, 0)$  under a dilation with a scale factor of  $\frac{1}{2}$  centered at the origin.

The rule is:

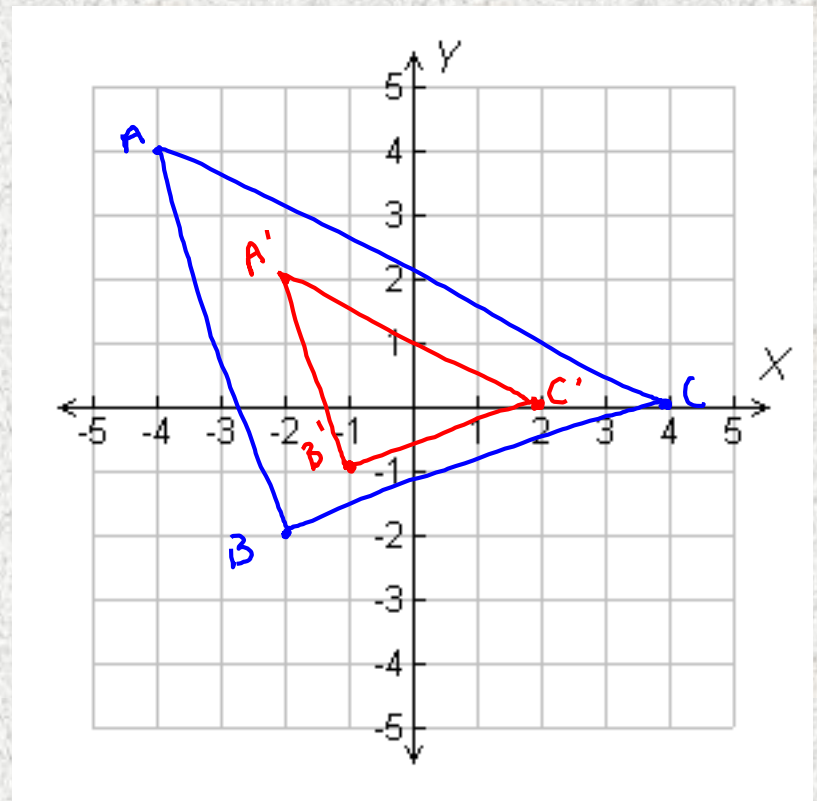
$$(x, y) \rightarrow (kx, ky)$$

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$$A(-4, 4) \rightarrow A'(-2, 2)$$

$$B(-2, -2) \rightarrow B'(-1, -1)$$

$$C(4, 0) \rightarrow C'(2, 0)$$



# EXAMPLES

4. A blueprint shows a reduction of a room that has a length of 10 feet and a width of 7 feet. In the blueprint, the room's length is 8 in. and its width is 5.6 in. If a desk is 2 feet wide, what is its width on the blueprint?

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# EXAMPLES

4. A blueprint shows a reduction of a room that has a length of 10 feet and a width of 7 feet. In the blueprint, the room's length is 8 in. and its width is 5.6 in. If a desk is 2 feet wide, what is its width on the blueprint? <sup>(24 in)</sup>

$$\text{SF: } \frac{\text{Length of room (blueprint)}}{\text{Length of room (actual)}} = \frac{8 \text{ in}}{10 \text{ ft}} = \frac{8 \text{ in}}{120 \text{ in}} = \frac{1}{15}$$

$$\frac{1}{15} = \frac{x \text{ (desk blueprint)}}{24 \text{ in (desk actual)}}$$

$$15x = 24$$

$$x = 1.6 \text{ in}$$

blueprint  
→  $\frac{\text{Length}}{\text{desk}} = \frac{\text{desk}}{\text{desk}}$

actual  
→  $\frac{\text{Length}}{\text{desk}}$

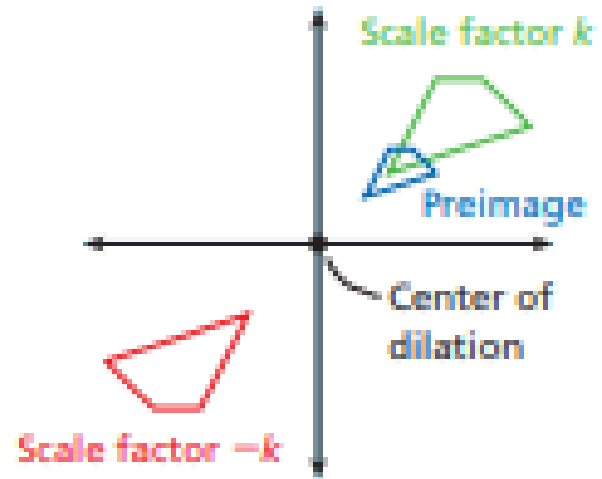
$$\frac{8 \text{ in}}{10 \text{ ft}} = \frac{x \text{ in}}{2 \text{ ft}}$$

$$10x = 16$$

$$x = 1.6 \text{ in}$$

# MORE INFORMATION

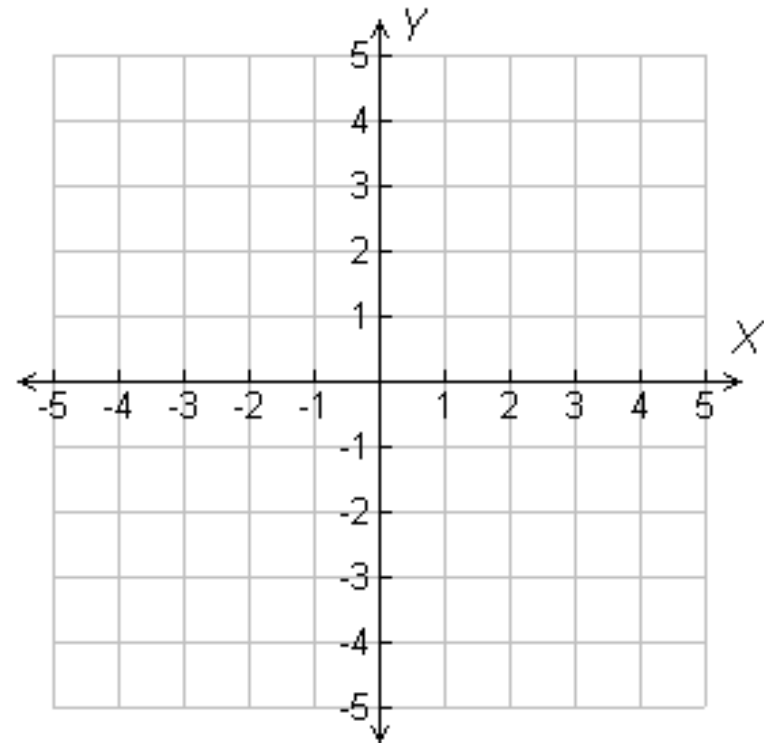
If the scale factor of a dilation is negative, the preimage is rotated by  $180^\circ$ . For  $k > 0$ , a dilation with a scale factor of  $-k$  is equivalent to the composition of a dilation with a scale factor of  $k$  that is rotated  $180^\circ$  about the center of dilation.



Our Geometry Standards do not require the use of the  $-k$ . Any problems that ask for a negative scale factor should be changed to positive.

# PRACTICE PROBLEM

5. Draw the image of the triangle with vertices  $A(1, -1)$ ;  $B(-2, -2)$ ;  $C(-1, 0)$  under a dilation with a scale factor of 2 centered at the origin.



# PRACTICE PROBLEM

5. Draw the image of the triangle with vertices  $A(1, -1)$ ;  $B(-2, -2)$ ;  $C(-1, 0)$  under a dilation with a scale factor of 2 centered at the origin.

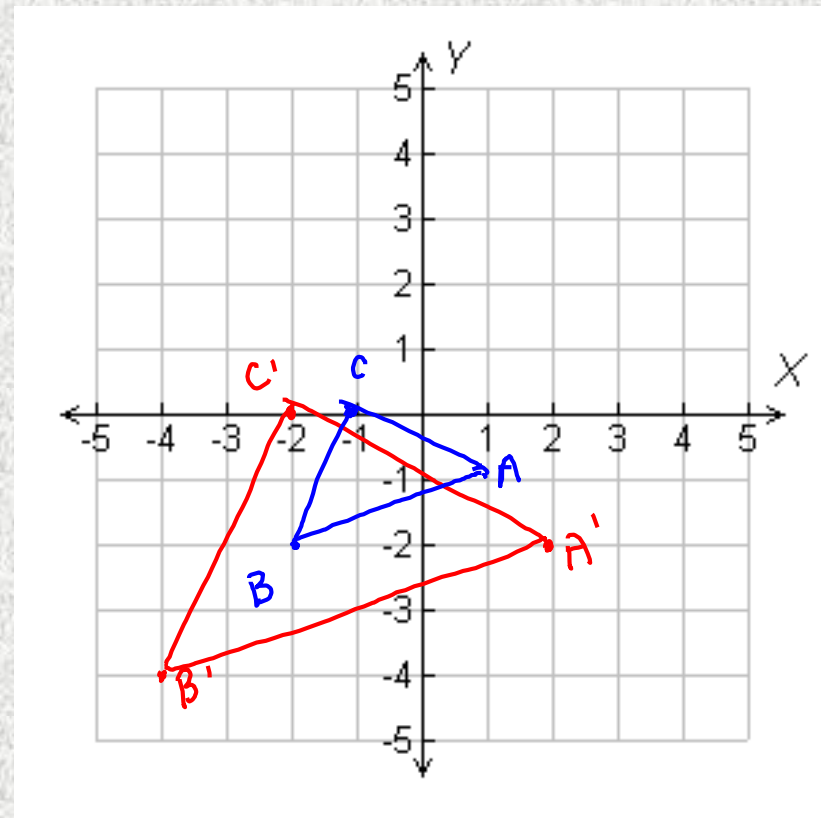
$$(x, y) \rightarrow (kx, ky)$$

$$(x, y) \rightarrow (2x, 2y)$$

$$A(1, -1) \rightarrow A'(2, -2)$$

$$B(-2, -2) \rightarrow B'(-4, -4)$$

$$C(-1, 0) \rightarrow C'(-2, 0)$$



# LEARNING RUBRIC

- ▶ Got It: Represents and applies to complex/real world situations
- ▶ Almost There: Make compound transformations on the coordinate plane
- ▶ Moving Forward: Make individual transformations on the coordinate plane
- ▶ Getting Started: Identify and sketch transformations

# HOMework

- ▶ 9–7 Pages 653–656: 4, 8, 9, 10, 16, 34, 36, 48