9-3/9-4

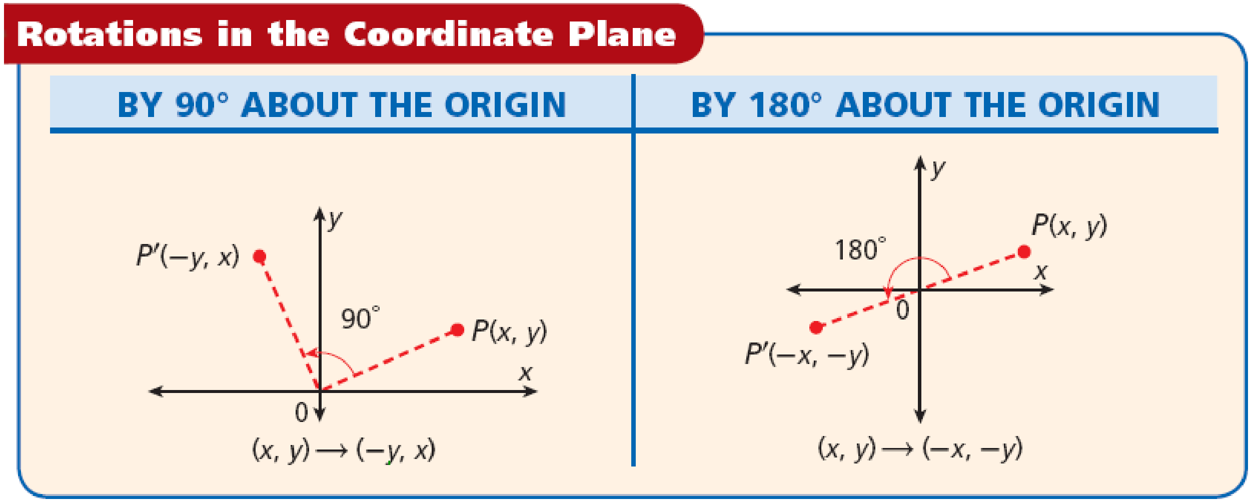
Rotations/Combinations

OBJECTIVES

- To identify and draw rotations
- To rotate figures on the coordinate plane
- To apply combinations of transformations on the coordinate plane

ROTATION

Rotation (turn) – transformation that moves around a point (center of rotation) by a certain degree measure (angle of rotation). This is a rigid motion.



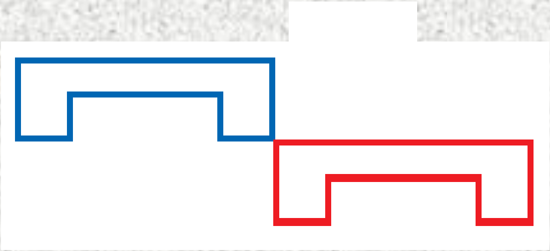
When no indication of direction is given, counterclockwise is understood. The rule for this is above. The rule for 90 degrees clockwise is $(x, y) \rightarrow (y, -x)$.

If the angle of a rotation in the coordinate plane is not a multiple of 90°, you can use sine and cosine ratios to find the coordinates of the image.

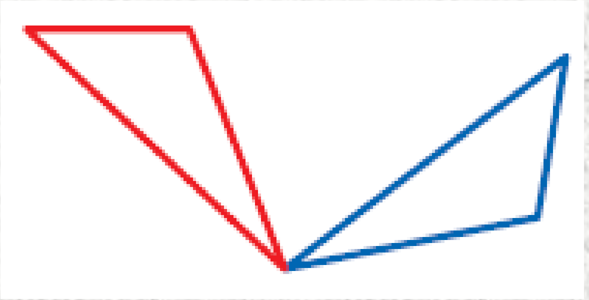
EXAMPLES

Tell whether each transformation appears to be a rotation. Explain.

1.



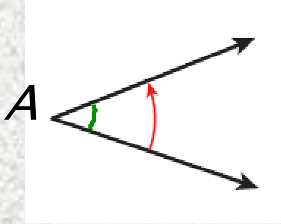
2.



No. The figure appears to be a translation.

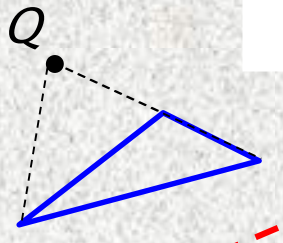
Yes. The image appears to be turned around a point.

EXAMPLES

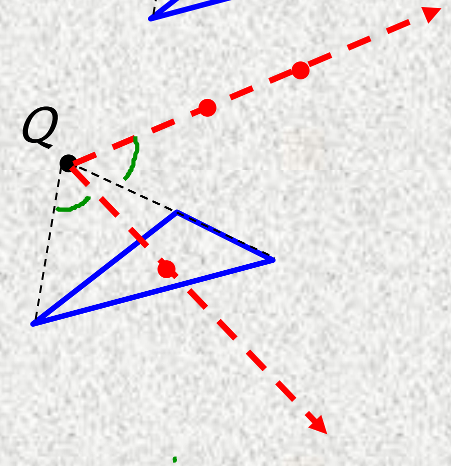


To draw a rotation:

Step 1. Draw a segment from each vertex to point of rotation.

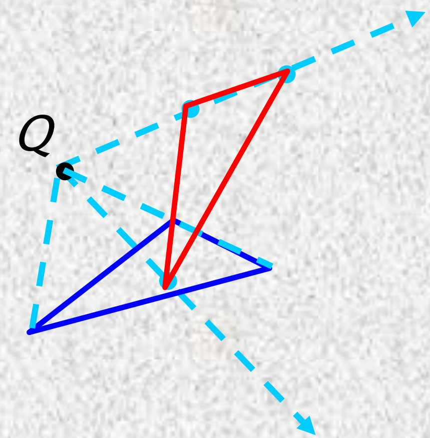


Step 2. Draw an angle congruent to angle of rotation onto each segment.



Step 3. Measure the distance from each vertex to point of rotation and mark off this distance on corresponding ray.

Step 4. Connect the images of the vertices.



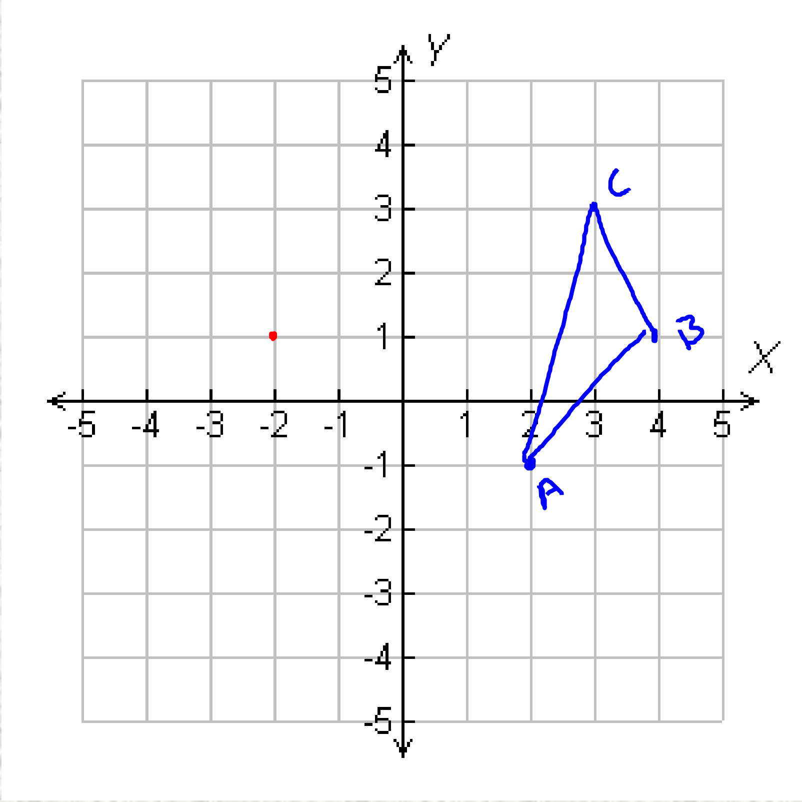
EXAMPLES

3. Rotate ΔABC by 180° about the origin.

A(2, –1); B(4, 1); C(3, 3)

The rule is:

$(x, y) \rightarrow (-x, -y)$



EXAMPLES

3. Rotate ΔABC by 180° about the origin.

A(2, −1); B(4, 1); C(3, 3)

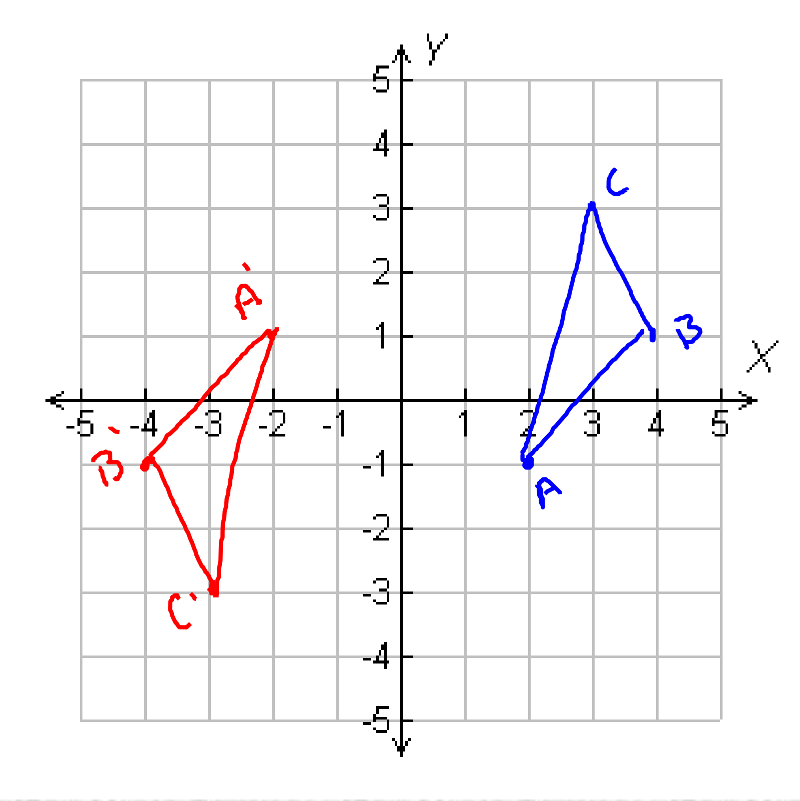
The rule is:

$(x, y) \rightarrow (-x, -y)$

A(2,-1) → A'(-2, 1)

B(4,1) → B'(-4,-1)

C(3,3) → C'(-3,-3)



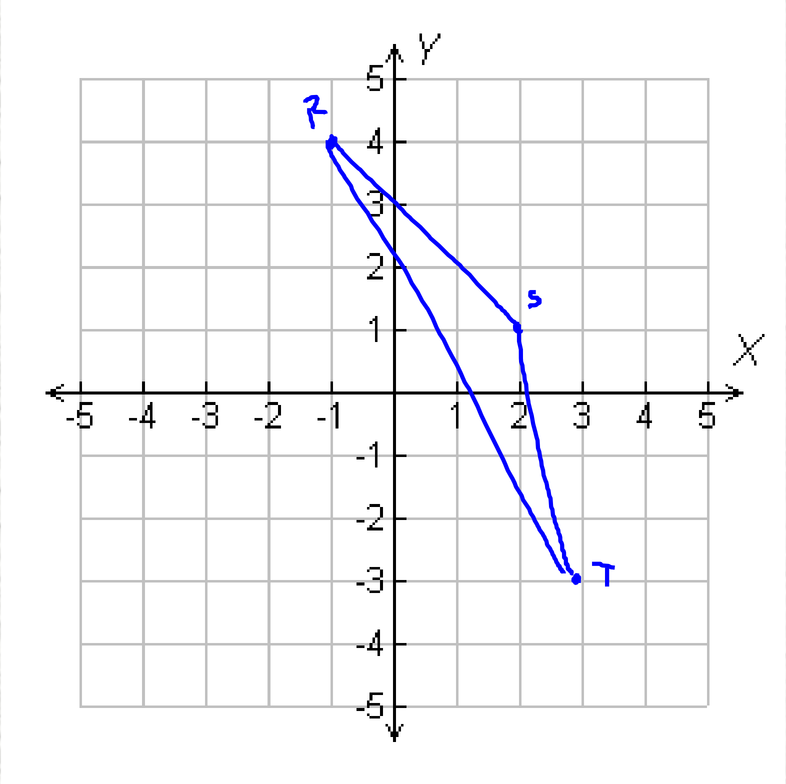
EXAMPLES

4. Rotate ΔRST by 90° about the origin.

R(−1, 4); S(2, 1); T(3, −3)

The rule is:

$(x, y) \rightarrow (-y, x)$



EXAMPLES

4. Rotate ΔRST by 90° about the origin.

R(−1, 4); S(2, 1); T(3, −3)

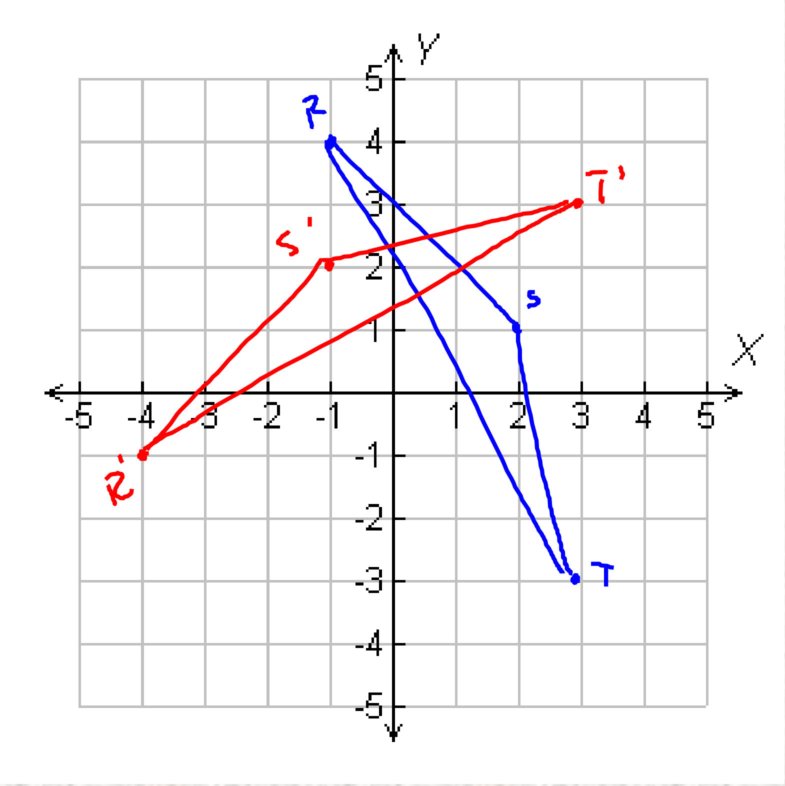
The rule is:

$(x, y) \rightarrow (-y, x)$

R(-1,4) → R'(-4,-1)

S(2,1) → S'(-1,2)

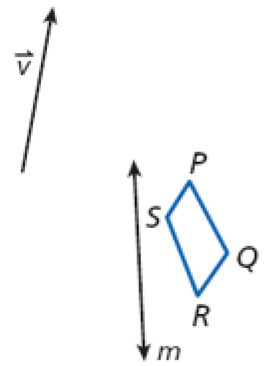
T(3,-3) → T'(3,3)



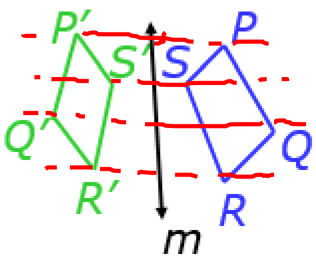
EXAMPLES

5. **Draw the result of the composition of isometries.**

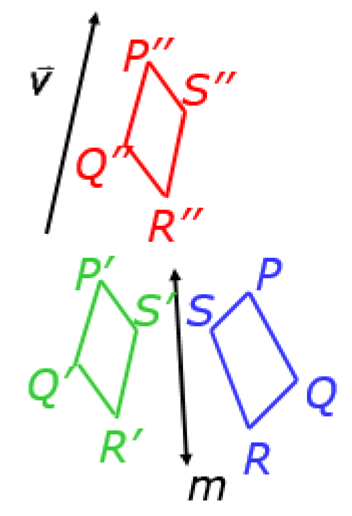
Reflect *PQRS* across line *m* and then translate it along $\vec{v}$.



Step 1 Draw *P′Q′R′S′*, the reflection image of *PQRS*.



Step 2 Translate *P′Q′R′S′* along $\vec{v}$ to find the final image, *P″Q″R″S″*.



EXAMPLES

6.

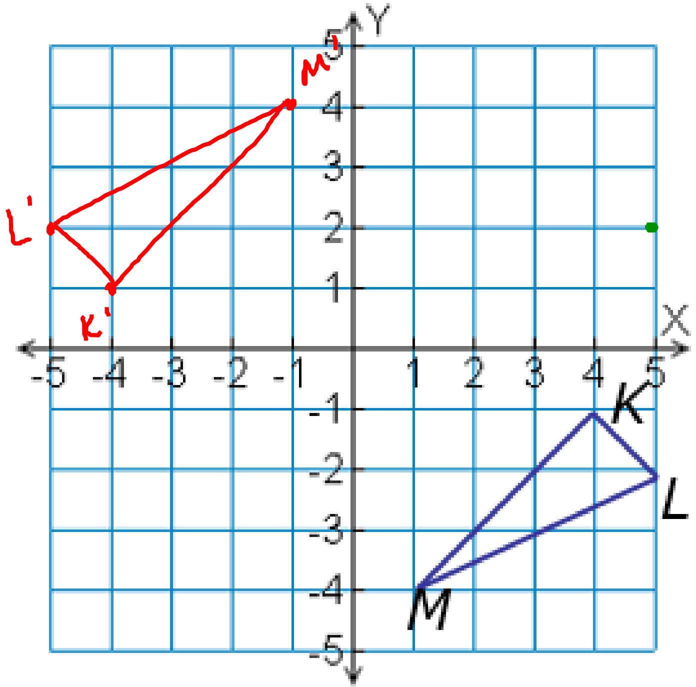
Δ*KLM* has vertices *K*(4, –1), *L*(5, –2), and *M*(1, –4). Rotate Δ*KLM* 180° about the origin and then reflect it across the *y*-axis.

$(x,y) \rightarrow (-x,-y)$

$K(4,-1) \rightarrow K'(-4,1)$

$L(5,-2) \rightarrow L'(-5,2)$

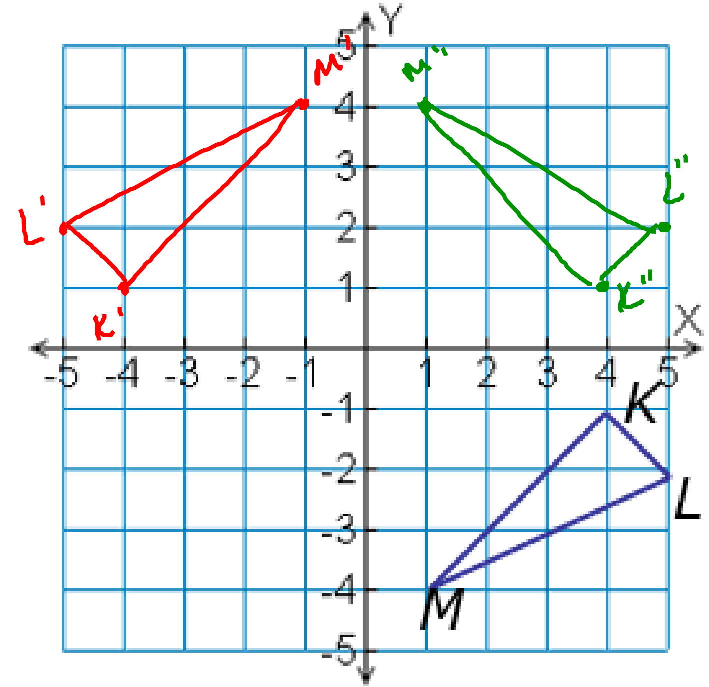
$m(1,-4) \rightarrow m'(-1,4)$



EXAMPLES

6.

Δ*KLM* has vertices *K*(4, –1), *L*(5, –2), and *M*(1, –4). Rotate Δ*KLM* 180° about the origin and then reflect it across the *y*-axis.



$(x,y) \to (-x,-y)$ $(x,y) \to (-x,y)$

$K(4,-1) \to K'(-4,1) \longrightarrow K''(4,1)$

$L(5,-2) \to L'(-5,2) \longrightarrow L''(5,2)$

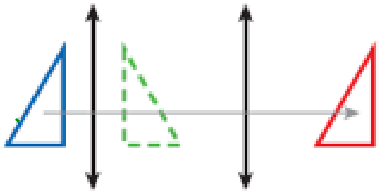
$m(1,-4) \to m'(-1,4) \longrightarrow m''(1,4)$

MORE INFORMATION

Theorem 12-4-2

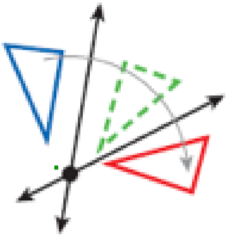
The composition of two reflections across two parallel lines is equivalent to a translation.

- The translation vector is perpendicular to the lines.
- The length of the translation vector is twice the distance between the lines.



The composition of two reflections across two intersecting lines is equivalent to a rotation.

- The center of rotation is the intersection of the lines.
- The angle of rotation is twice the measure of the angle formed by the lines.



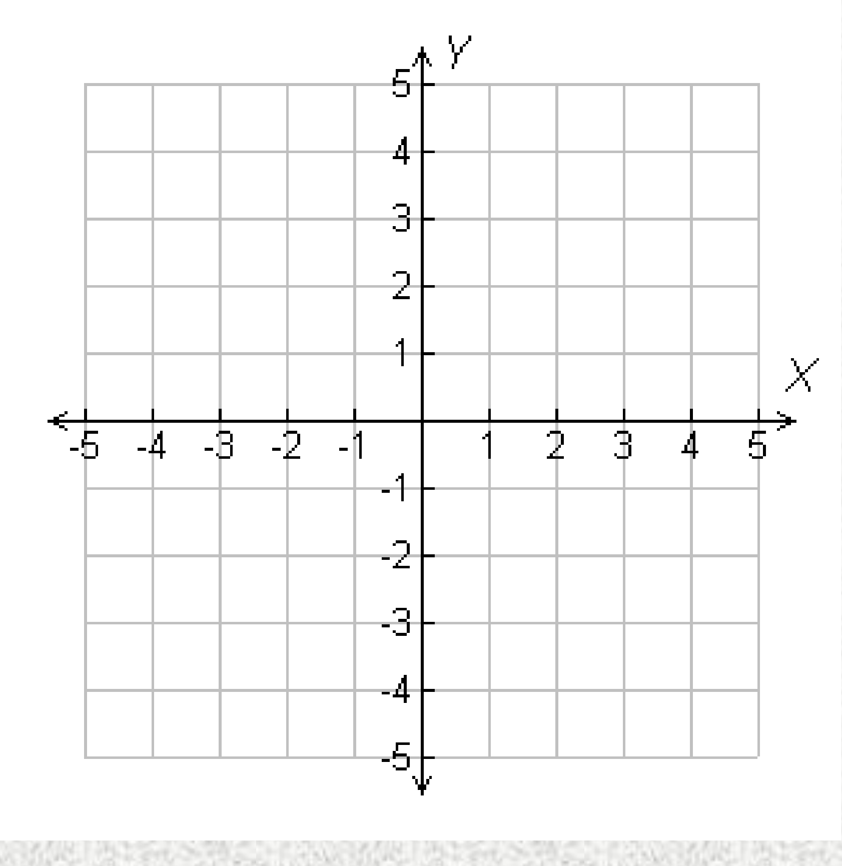
Theorem 12-4-3

Any translation or rotation is equivalent to a composition of two reflections.

PRACTICE PROBLEM

7. Reflect ΔRST across the line y = x, then rotate it 90˚ about the origin.

R(5, −2); S(1, −4); T(−3, 3)



PRACTICE PROBLEM

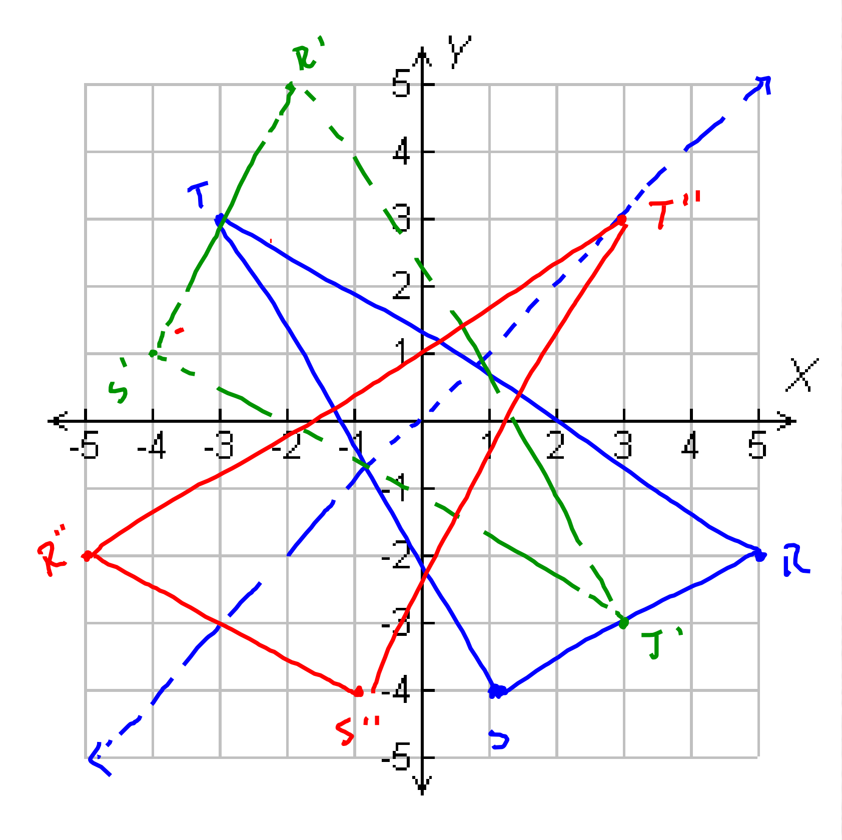
7. Reflect ΔRST across the line y = x, then rotate it 90° about the origin.

R(5, –2); S(1, –4); T(–3, 3)

R' (-2, 5) → R'' (-5, -2)

S' (-4, 1) → S'' (-1, -4)

T' (3, -3) → T'' (3, 3)



LEARNING RUBRIC

- Got It: Represents and applies to complex/real world situations
- Almost There: Make compound transformations on the coordinate plane
- Moving Forward: Make individual transformations on the coordinate plane
- Getting Started: Identify and sketch transformations

HOMEWORK

- 9-3 Pages 622-625: 12,18,20,38,40,42,44
- 9-4 Pages 629-631: 4,10,12,16,18,23,24