7-6

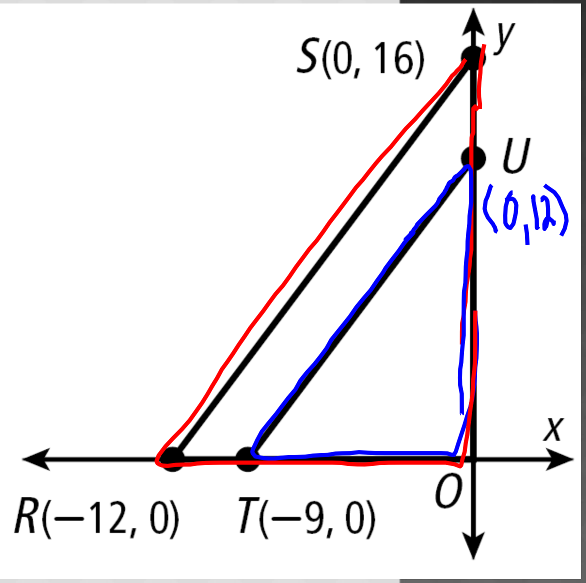
Similarity on Coordinate Plane

OBJECTIVE

- To prove similarity with distance formula

Example 2: Finding Coordinates of Similar Triangles

Given that Δ*TUO* ~ Δ*RSO*, find the coordinates of *U* and the scale factor.



Since Δ*TUO* ~ Δ*RSO*,

$$\frac{RO}{TO} = \frac{OX}{OU}$$

$$\frac{12}{9} = \frac{16}{OU}$$ *Substitute 12 for RO, 9 for TO, and 16 for OS.*

$12OU = 144$ *Cross Products Prop.*

$OU = 12$ *Divide both sides by 12.*

$$(0,16) \longrightarrow \left(0 \bullet \frac{3}{4}, 16 \bullet \frac{3}{4}\right) \longrightarrow (0,12)$$

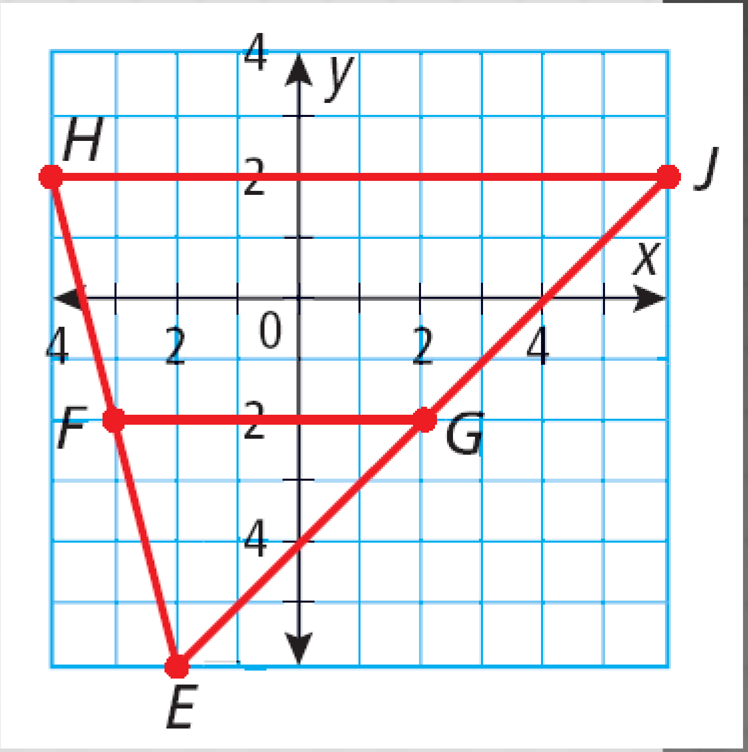
So the scale factor is $\frac{3}{4}$.

Example 3: Proving Triangles Are Similar

Given: ***E*(–2, –6), *F*(–3, –2), *G*(2, –2), *H*(–4, 2), and *J*(6, 2).**

Prove: Δ*EHJ* ~ Δ*EFG*.

Step 1 Plot the points and draw the triangles.



Example 3 Continued

Step 2 Use the Distance Formula to find the side lengths.

$$EH = \sqrt{[-4-(-2)]^2 + [2-(-6)]^2} = \sqrt{68} = 2\sqrt{17}$$

$$EJ = \sqrt{[6-(-2)]^2 + [2-(-6)]^2} = \sqrt{128} = 8\sqrt{2}$$

$$EF = \sqrt{[-3-(-2)]^2 + [-2-(-6)]^2} = \sqrt{17}$$

$$EG = \sqrt{[2-(-2)]^2 + [-2-(-6)]^2} = \sqrt{32} = 4\sqrt{2}$$

Example 3 Continued

Step 3 Find the similarity ratio.

$$\frac{EH}{EF} = \frac{2\sqrt{17}}{\sqrt{17}} = \frac{2}{1} = 2$$

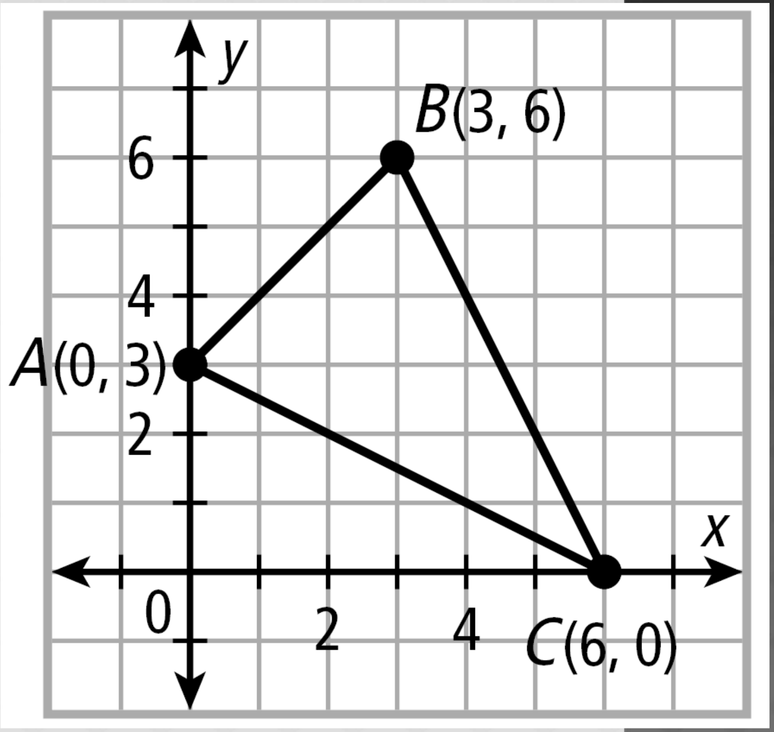
$$\frac{EJ}{EF} = \frac{8\sqrt{2}}{4\sqrt{2}} = \frac{4}{2} = 2$$

Since $\frac{EH}{EF} = \frac{EJ}{EG}$ and $\angle E \cong \angle E$, by the Reflexive Property, $\Delta EHJ \sim \Delta EFG$ by SAS ~ .

Example 4: Using the SSS Similarity Theorem

Graph the image of Δ*ABC* after a dilation with scale factor $\frac{2}{3}$.

Verify that Δ*A'B'C'* ~ Δ*ABC*.



Example 4 Continued

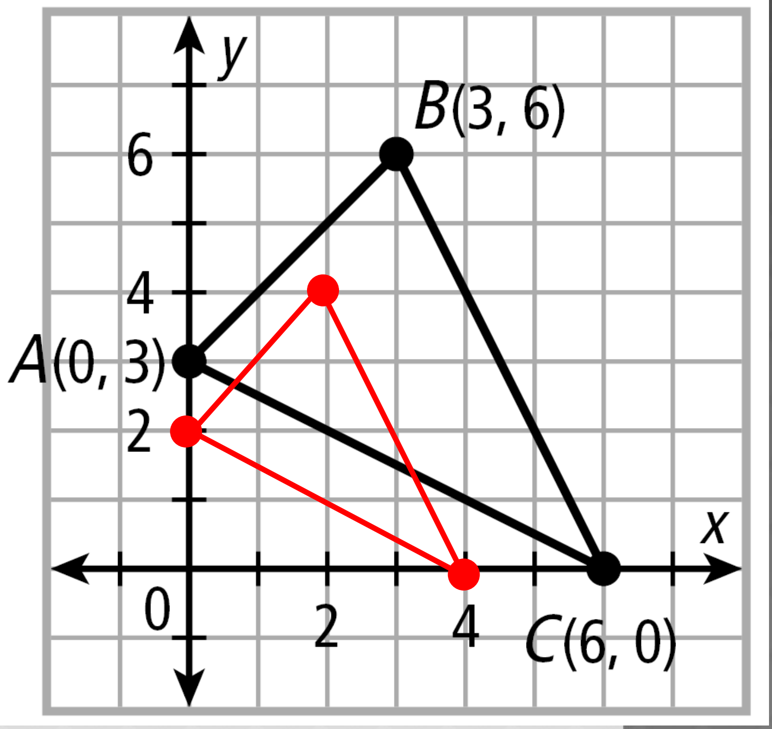
Step 1 Multiply each coordinate by $\frac{2}{3}$ to find the coordinates of the vertices of $\Delta A'B'C'$.

$$A(0, 3) \longrightarrow A'\left(0 \bullet \frac{2}{3}, 3 \bullet \frac{2}{3}\right) \longrightarrow A'(0, 2)$$

$$B(3, 6) \longrightarrow B'\left(3 \bullet \frac{2}{3}, 6 \bullet \frac{2}{3}\right) \longrightarrow B'(2, 4)$$

$$C(6, 0) \longrightarrow C'\left(6 \bullet \frac{2}{3}, 0 \bullet \frac{2}{3}\right) \longrightarrow C'(4, 0)$$

Step 2 Graph $\Delta A'B'C'$.



Example 4 Continued

Step 3 Use the Distance Formula to find the side lengths.

$$AB = \sqrt{(3-0)^2 + (6-3)^2} = \sqrt{18} = 3\sqrt{2}$$

$$A'B' = \sqrt{(2-0)^2 + (4-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(6-3)^2 + (0-6)^2} = \sqrt{45} = 3\sqrt{5}$$

$$B'C' = \sqrt{(4-2)^2 + (0-4)^2} = \sqrt{20} = 2\sqrt{5}$$

$$AC = \sqrt{(6-0)^2 + (0-3)^2} = \sqrt{45} = 3\sqrt{5}$$

$$A'C' = \sqrt{(4-0)^2 + (0-2)^2} = \sqrt{20} = 2\sqrt{5}$$

Example 4 Continued

Step 4 Find the similarity ratio.

$$\frac{A'B'}{AB} = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}, \frac{B'C'}{BC} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}, \frac{A'C'}{AC} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}$$

Since $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$, $\Delta ABC \sim \Delta A'B'C'$ by SSS ~.

HOMEWORK

Pages 513 - 514

10, 12, 14, 16, 22