

**6-3**

**PROVING PARALLELOGRAMS**

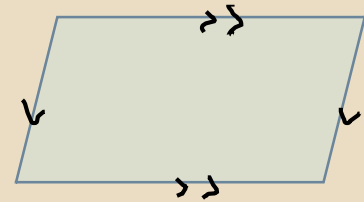


# OBJECTIVE

TO DETERMINE  
WHETHER A  
QUADRILATERAL  
IS A  
PARALLELOGRAM

# KEY CONCEPTS

To prove that a quadrilateral is a parallelogram, you can use the definition of parallelogram or any of the Theorems that follow.



both opp sides  $\parallel \rightarrow \square$

## Theorems

## Conditions for Parallelograms

THEOREM	EXAMPLE
<p><b>6-3-1</b> If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides <math>\parallel</math> and <math>\cong \rightarrow \square</math>)</p>	
<p><b>6-3-2</b> If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides <math>\cong \rightarrow \square</math>)</p>	
<p><b>6-3-3</b> If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. <math>\sphericalangle \cong \rightarrow \square</math>)</p>	

# KEY CONCEPTS

The two theorems below can also be used to show that a given quadrilateral is a parallelogram.

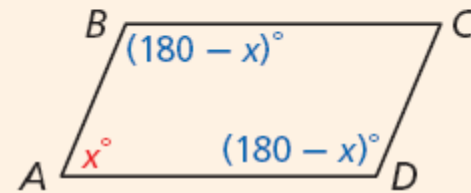
## Theorems

## Conditions for Parallelograms

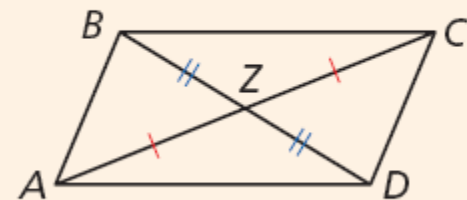
### THEOREM

### EXAMPLE

**6-3-4** If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.  
(quad. with  $\angle$  supp. to cons.  $\angle$   $\rightarrow$   $\square$ )



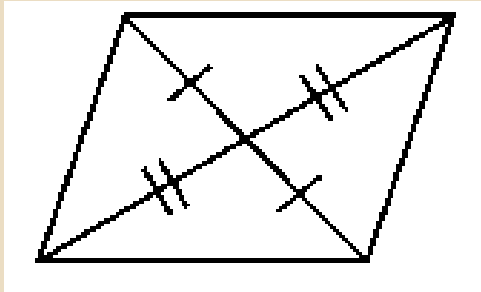
**6-3-5** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.  
(quad. with diags. bisecting each other  $\rightarrow$   $\square$ )



# CLASS WORK

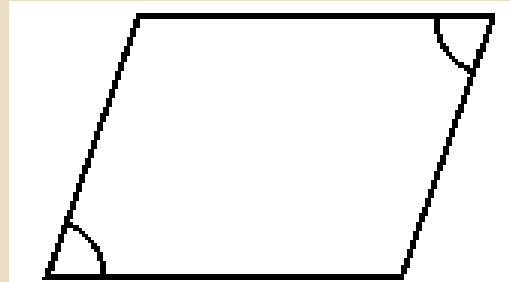
Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

1.



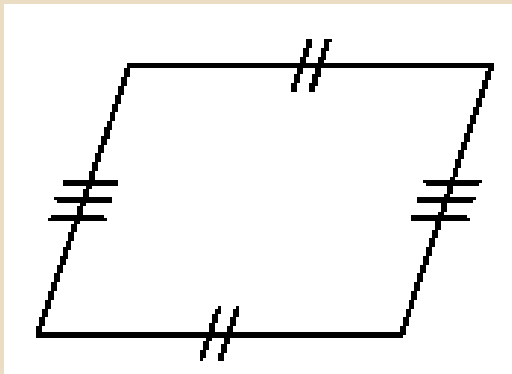
Yes. Both diagonals bisect each other.

2.



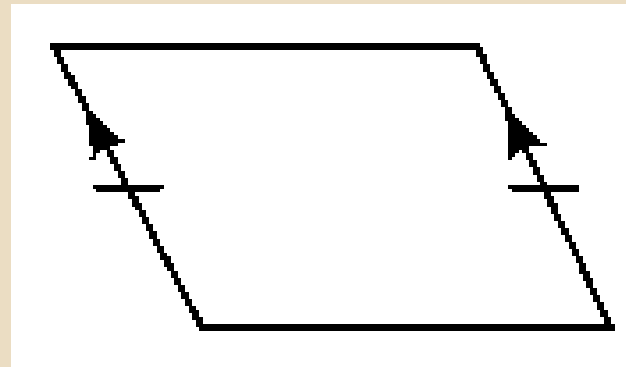
No. Only one set of opposite angles marked congruent.

3.



Yes. Both sets of opposite sides congruent.

4.

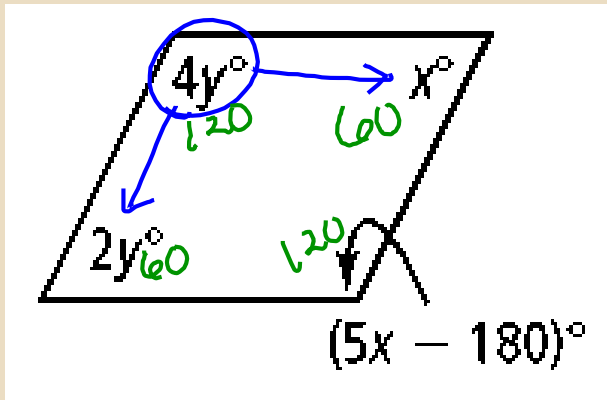


Yes. Same set of opposite sides both parallel and congruent.

# CLASS WORK

For what values of  $x$  and  $y$  must the figure be a parallelogram?

5.



$$2y + 4y = 180$$

$$6y = 180$$

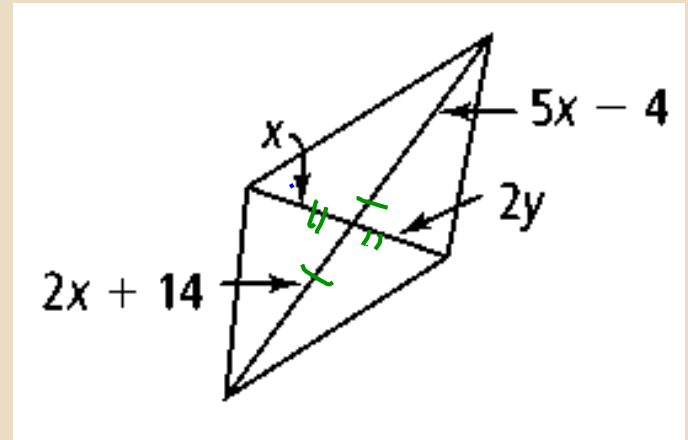
$$y = 30$$

$$4y + x = 180$$

$$120 + x = 180$$

$$x = 60$$

6.



$$x = 2y$$

$$6 = 2y$$

$$3 = y$$

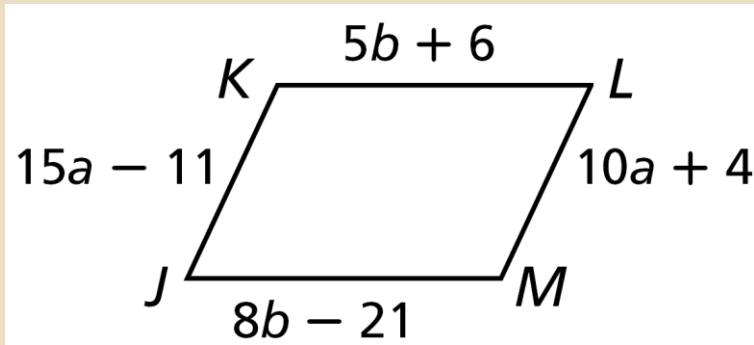
$$2x + 14 = 5x - 4$$

$$18 = 3x$$

$$6 = x$$

# CLASS WORK

7. Show that JKLM is a parallelogram for  $a = 3$  and  $b = 9$ .



$$15a - 11 = 10a + 4$$

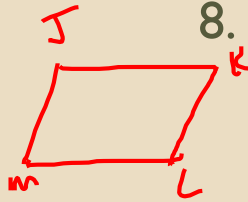
$$15(3) - 11 = 10(3) + 4$$

$$34 = 34 \checkmark$$

$$8b - 21 = 5b + 6$$

$$3b = 27$$

$$b = 9 \checkmark$$



8. Show that quadrilateral JKLM is a parallelogram by using the definition of parallelogram.

$J(-1, -6), K(-4, -1), L(4, 5), M(7, 0)$ .

$$\overline{JK}: m = \frac{-1 - (-6)}{-4 - (-1)} = \frac{5}{-3}$$

$$\overline{ML}: m = \frac{0 - 5}{7 - 4} = -\frac{5}{3} \checkmark$$

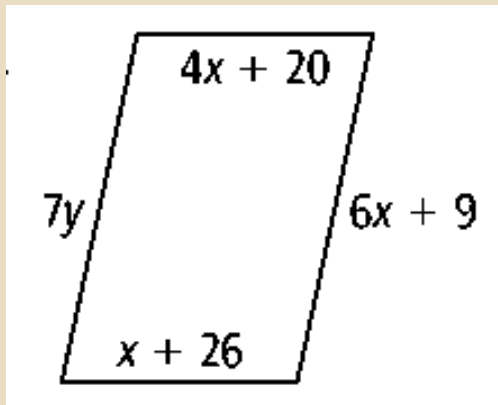
$$\overline{MJ}: m = \frac{0 - (-6)}{7 - (-1)} = \frac{6}{8} = \frac{3}{4} \checkmark$$

$$\overline{KL}: m = \frac{5 - (-1)}{4 - (-4)} = \frac{6}{8} = \frac{3}{4} \checkmark$$

# EXIT PROBLEMS

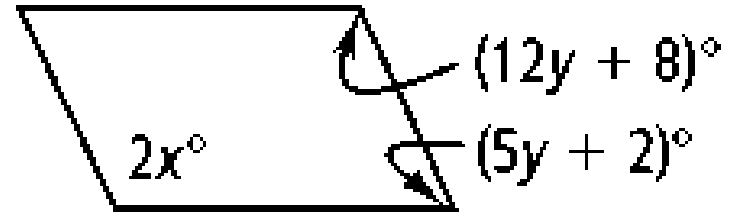
For what values of  $x$  and  $y$  must the figure be a parallelogram?

9.



$$\begin{aligned}4x + 20 &= x + 26 & 7y &= 6x + 9 \\3x &= 6 & 7y &= 6(2) + 9 \\x &= 2 & 7y &= 21 \\ & & y &= 3\end{aligned}$$

10.



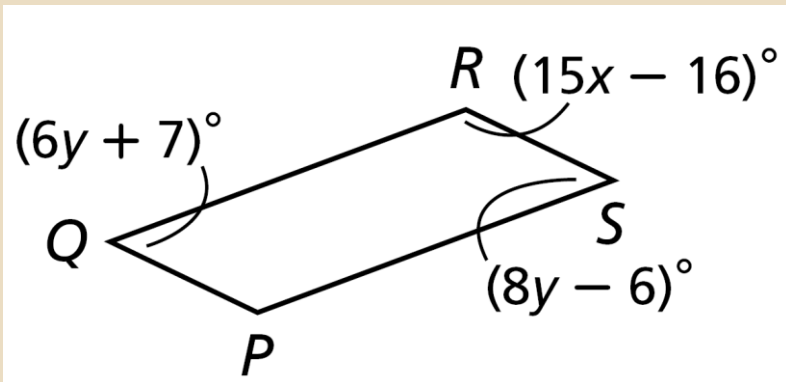
$$\begin{aligned}5y + 2 + 12y + 8 &= 180 \\17y + 10 &= 180 \\17y &= 170 \\y &= 10\end{aligned}$$

$$\begin{aligned}5y + 2 + 2x &= 180 \\5(10) + 2 + 2x &= 180 \\52 + 2x &= 180 \\2x &= 128 \\x &= 64\end{aligned}$$



# EXIT PROBLEMS

11. Show that  $PQRS$  is a parallelogram for  $x = 10$  and  $y = 6.5$ .



$$\begin{aligned}
 15x - 16 + 6y + 7 &= 180 \\
 15(10) - 16 + 6(6.5) + 7 &= 180 \\
 150 - 16 + 39 + 7 &= 180 \\
 180 &= 180
 \end{aligned}$$

$$\begin{aligned}
 15x - 16 + 8y - 6 &= 180 \\
 15(10) - 16 + 8(6.5) - 6 &= 180 \\
 150 - 16 + 52 - 6 &= 180 \\
 180 &= 180
 \end{aligned}$$

12. Use Theorem 6-3-1 (pair of opposite sides both parallel and congruent) to show that the quadrilateral with vertices  $K(-3, 0)$ ,  $L(-5, 7)$ ,  $M(3, 5)$ , and  $N(5, -2)$  is a parallelogram.

$$\begin{aligned}
 \overline{KL}: m &= \frac{7 - 0}{-5 + 3} = -\frac{7}{2} \\
 KL &= \sqrt{(-5 + 3)^2 + (7 - 0)^2} \\
 &= \sqrt{4 + 49} = \sqrt{51}
 \end{aligned}$$

$$\begin{aligned}
 \overline{MN}: m &= \frac{-2 - 5}{5 - 3} = -\frac{7}{2} \\
 MN &= \sqrt{(5 - 3)^2 + (-2 - 5)^2} \\
 &= \sqrt{4 + 49} = \sqrt{51}
 \end{aligned}$$

# LEARNING RUBRIC

Got It: Completes general proofs and uses proof to prove parallelograms

Almost There: Uses formulas to prove parallelograms on the coordinate plane

Moving Forward: Applies the properties of parallelograms to find or check given values of variables that prove parallelograms

Getting Started: Identifies correctly marked diagrams that prove parallelograms

# HOMework

Pages 414 – 417

10 – 22 even

26, 27, 34, 36

# SUMMARY

## A QUADRILATERAL IS A PARALLELOGRAM IF:

- ❖ PROVE THAT BOTH SETS OF OPPOSITE SIDES ARE PARALLEL.
- ❖ PROVE THAT BOTH SETS OF OPPOSITE SIDES ARE CONGRUENT.
- ❖ PROVE THAT AN ANGLE IS SUPPLEMENTARY TO BOTH OF ITS CONSECUTIVE ANGLES.
- ❖ PROVE THAT BOTH PAIRS OF OPPOSITE ANGLES ARE CONGRUENT.
- ❖ PROVE THAT THE DIAGONALS BISECT EACH OTHER.
- ❖ PROVE THAT ONE PAIR OF OPPOSITE SIDES IS PARALLEL AND CONGRUENT.