5-5

Indirect Proof and Inequalities in One Triangle

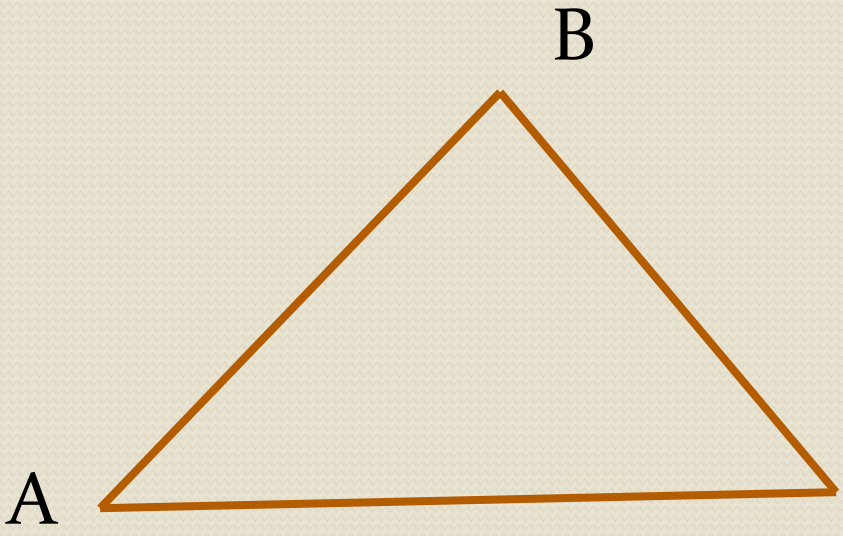
OBJECTIVE

- To write indirect proofs
- To apply inequalities in one triangle

KEY CONCEPT

Theorem 5-5-1:

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

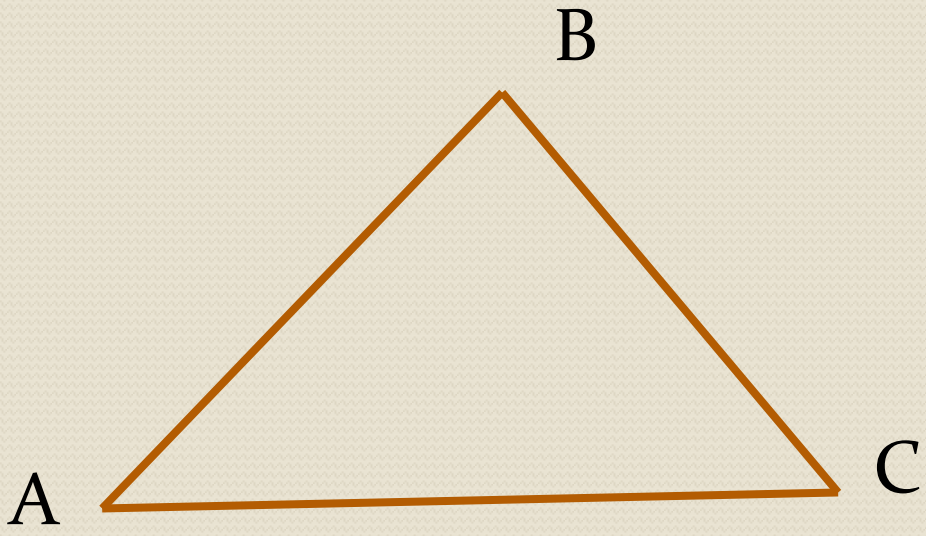


If $AB > BC$,
then $m\angle C > m\angle A$

KEY CONCEPT

Theorem 5-5-2:

If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.



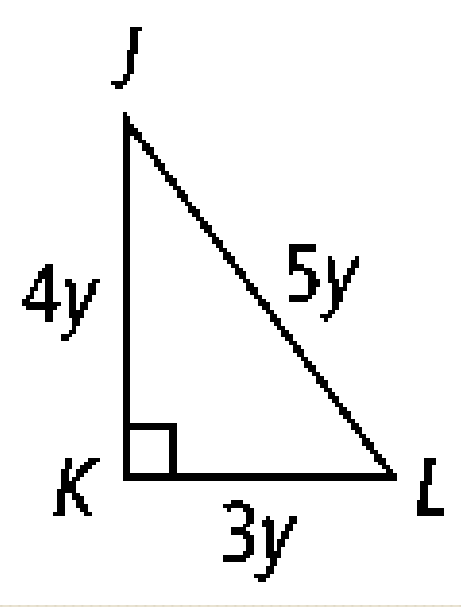
If $m\angle B > m\angle A$, then $AC > BC$

CLASS WORK

1. List the sides of each triangle in order from shortest to longest.

   ΔABC, with $m\angle A = 122$, $m\angle B = 22$, and $m\angle C = 36$

2. List the angles from largest to smallest.



CLASS WORK

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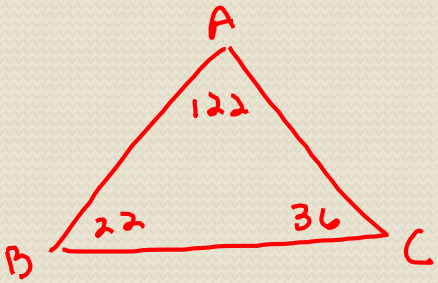
ΔABC, with $m\angle A = 122$, $m\angle B = 22$, and $m\angle C = 36$

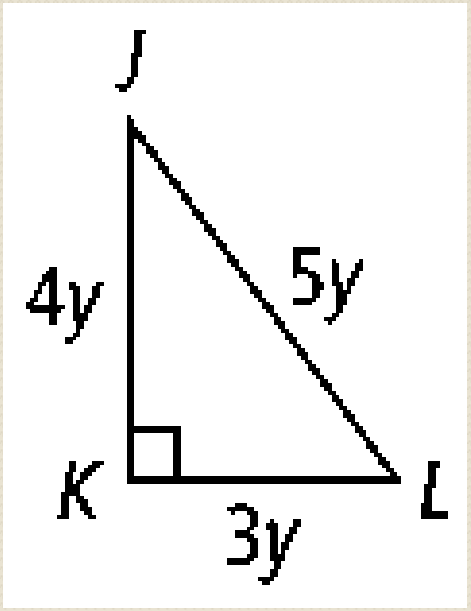


$22 < 36 < 122$

$\overline{AC}, \overline{AB}, \overline{BC}$

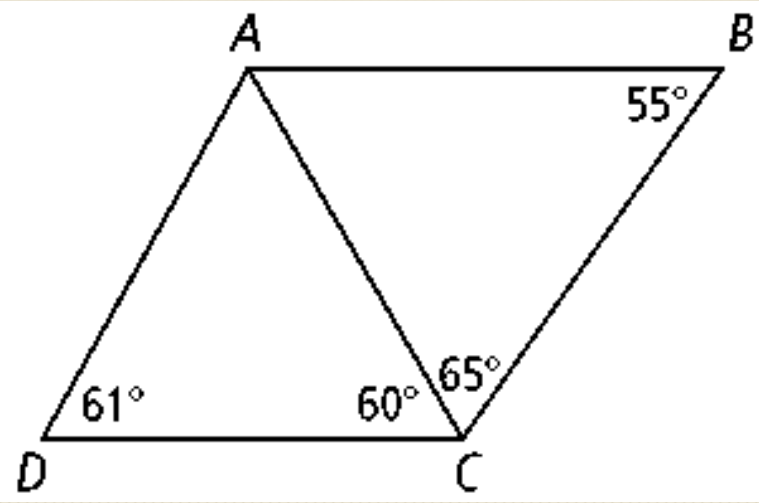
2. List the angles from largest to smallest.

$\angle K, \angle L, \angle J$



CLASS WORK

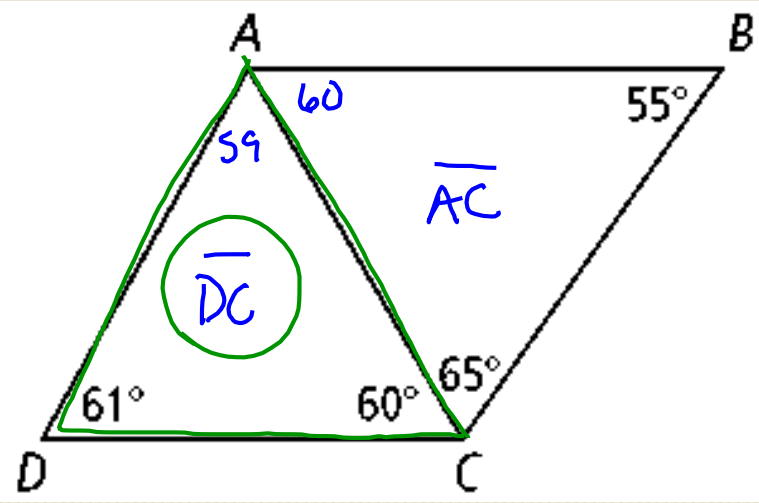
3. Determine which side is shortest in the diagram.



4. List the sides in order from shortest to longest in ΔPQR, with $m\angle P = 45$, $m\angle Q = 10x + 30$, and $m\angle R = 5x$.

CLASS WORK

3. Determine which side is shortest in the diagram.



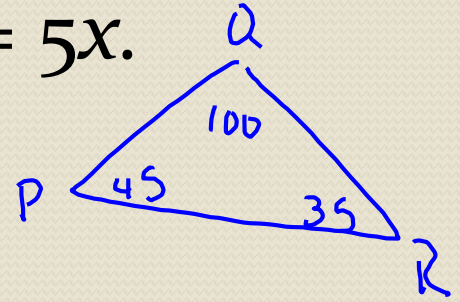
4. List the sides in order from shortest to longest in ΔPQR, with $m\angle P = 45$, $m\angle Q = 10x + 30$, and $m\angle R = 5x$.

$45 + 10x + 30 + 5x = 180$

$15x + 75 = 180$

$15x = 105$

$x = 7$



$\overline{PQ}, \overline{QR}, \overline{PR}$

KEY CONCEPT

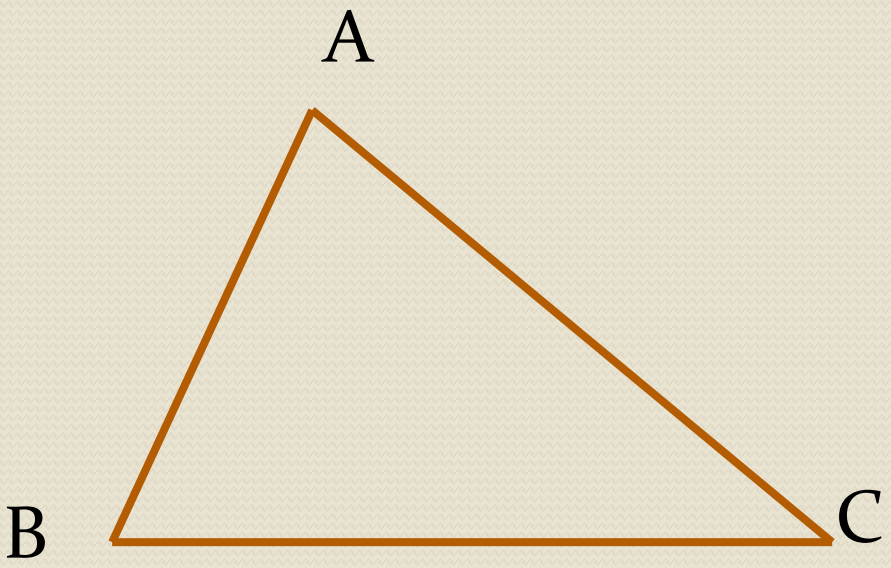
Triangle Inequality Theorem:

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AB + BC > AC$

$BC + AC > AB$

$AB + AC > BC$



CLASS WORK

Can a triangle have sides with the given lengths? Explain

5. 8 cm, 7 cm, 9 cm
6. 7 ft, 13 ft, 6 ft

The lengths of two sides of a triangle are given. Describe the possible lengths for the third side.

7. 5, 11
8. 12, 12

CLASS WORK

Can a triangle have sides with the given lengths? Explain

5. 8 cm, 7 cm, 9 cm $7+8>9$ yes
6. 7 ft, 13 ft, 6 ft $7+6 \not> 13$ no

The lengths of two sides of a triangle are given. Describe the possible lengths for the third side.

7. 5, 11, x $x+5>11$ $x>6$ $5+11>x$ $16>x$ $6<x<16$
8. x, 12, 12 $x+12>10$ $x>0$ $12+12>x$ $24>x$ $0<x<24$

5-6

INEQUALITIES IN TWO TRIANGLES

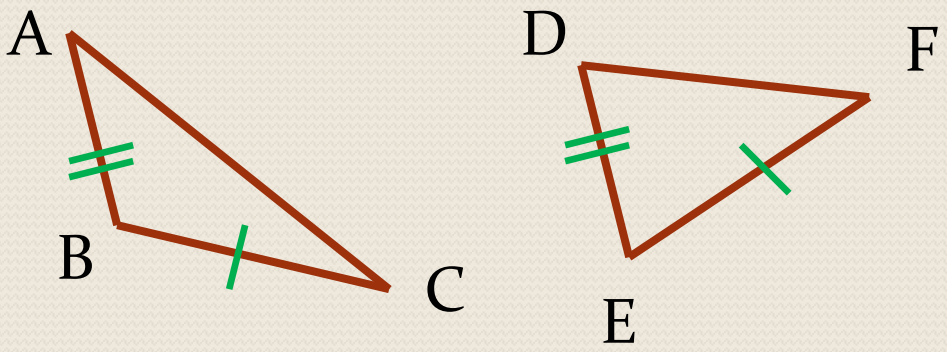
OBJECTIVE

To apply inequalities in two triangles

KEY CONCEPT

The Hinge Theorem: (SAS Inequality Theorem)

If two sides of one triangle are congruent to two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

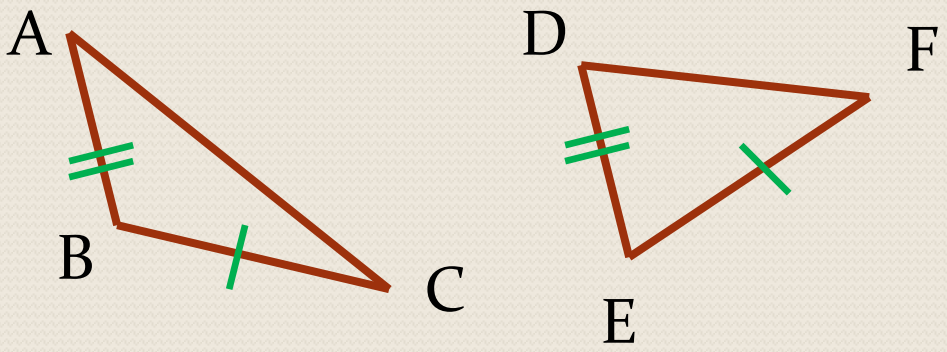


If $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$ and $m\angle B > m\angle E$, then AC > DF.

KEY CONCEPT

Converse of the Hinge Theorem: (SSS Inequality)

If two sides of one triangle are congruent to two sides of another triangle, and the third sides are not congruent, then the larger included angle is opposite the longer third side.

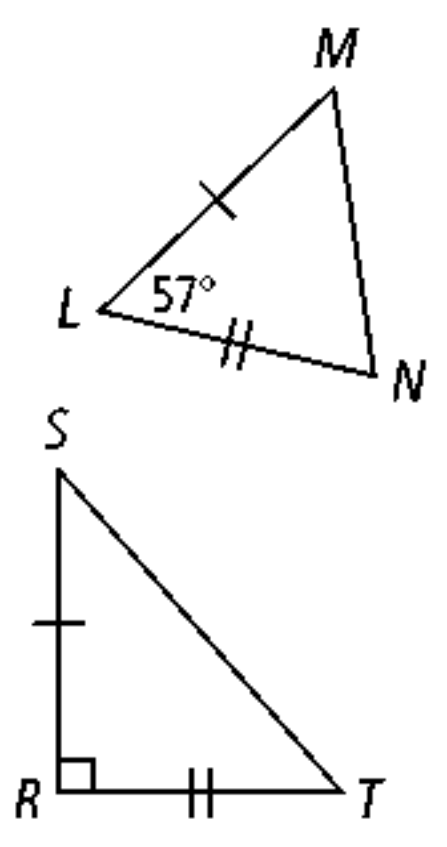


If $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$ and AC > DF, then $m\angle B > m\angle E$.

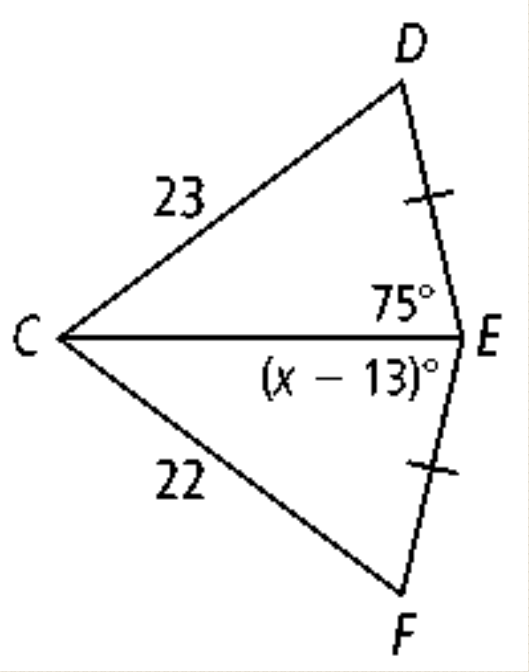
CLASS WORK

1. Write an inequality relating the given side lengths.

ST and *MN*



2. Find the range of possible values for each variable.



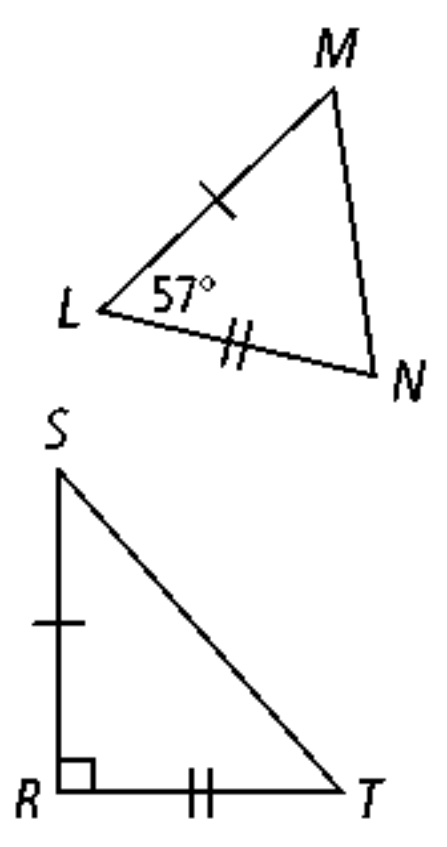
CLASS WORK

1. Write an inequality relating the given side lengths.

ST and *MN*

90 > 57

ST > MN



2. Find the range of possible values for each variable.

22 < 23

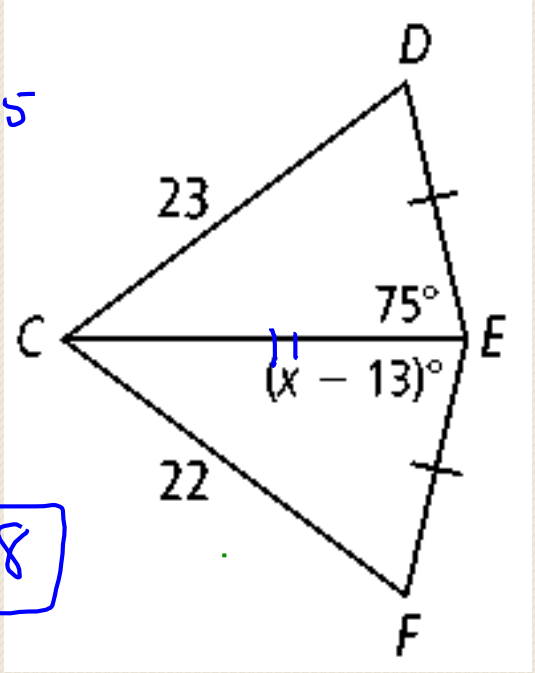
x − 13 < 75

x < 88

x − 13 > 0

x > 13

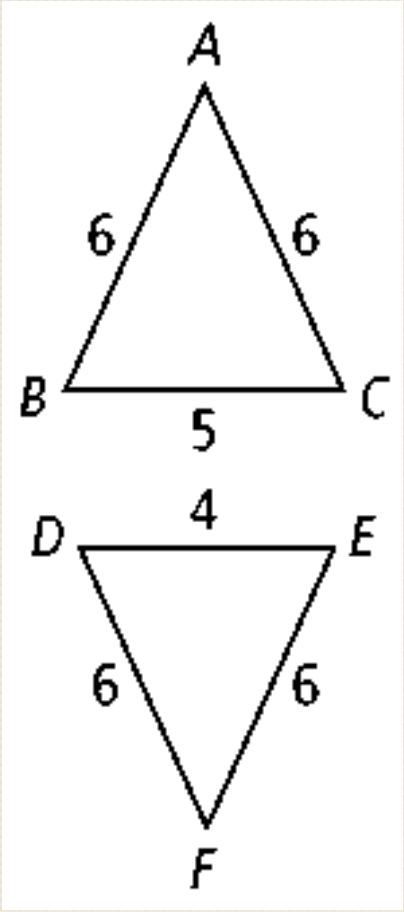
13 < x < 88



CLASS WORK

Write an inequality relating the given angle measures. If there is not enough information to reach a conclusion, write *no conclusion*.

3. $m\angle A$ and $m\angle F$



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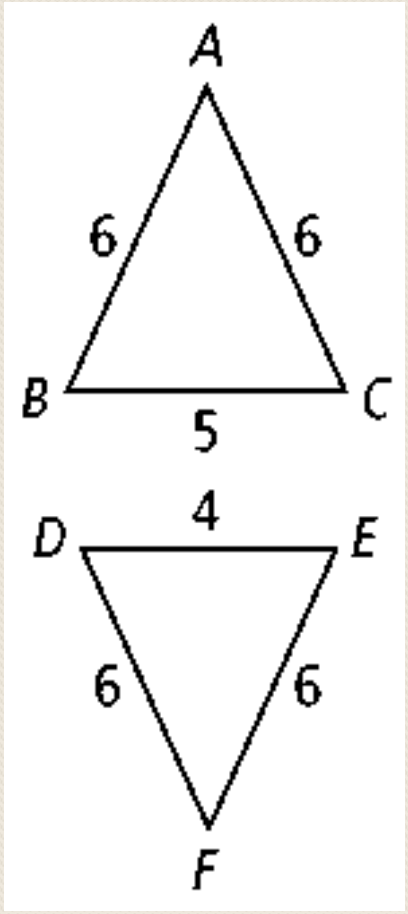
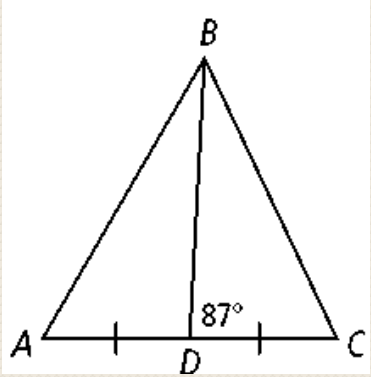


$m\angle A > m\angle F$

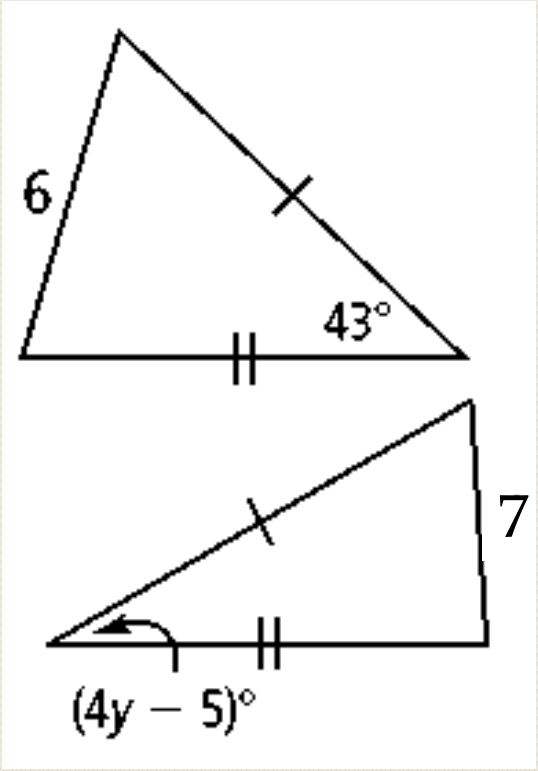
CLASS WORK

5. Write an inequality relating the given side lengths.

BA and *BC*



6. Find the range of possible values for each variable.



CLASS WORK

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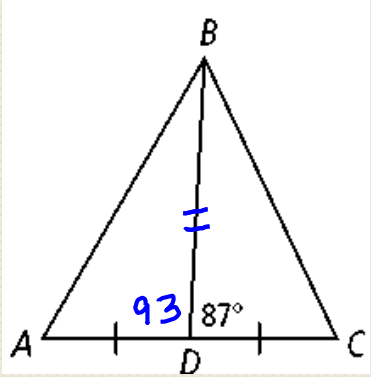
BA and *BC*



BA > BC

7 > 6

4y − 5 > 43

4y > 48

y > 12

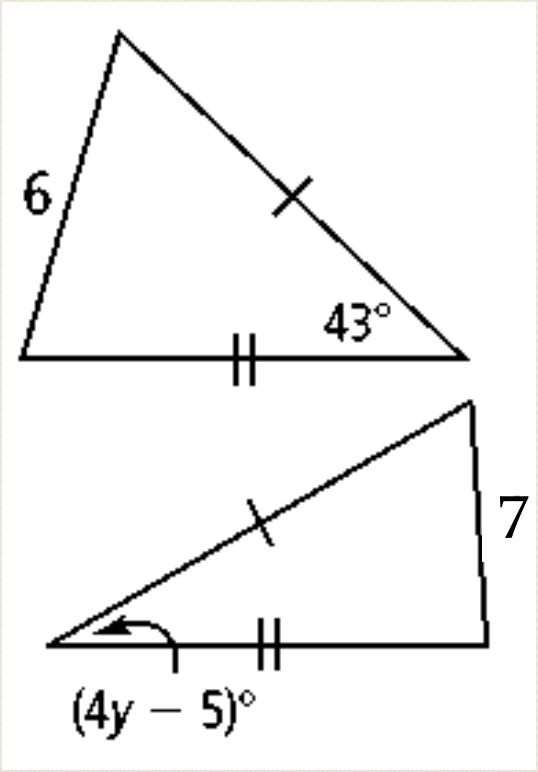
4y − 5 < 180

4y < 185

y < 46.25

6. Find the range of possible values for each variable.

12 < x < 46.25



CLASS WORK

4. A crocodile opens his jaws at a 30° angle. He closes his jaws, then opens them again at a 36° angle. In which case is the distance between the tip of his upper jaw and the tip of his lower jaw greater? Explain.

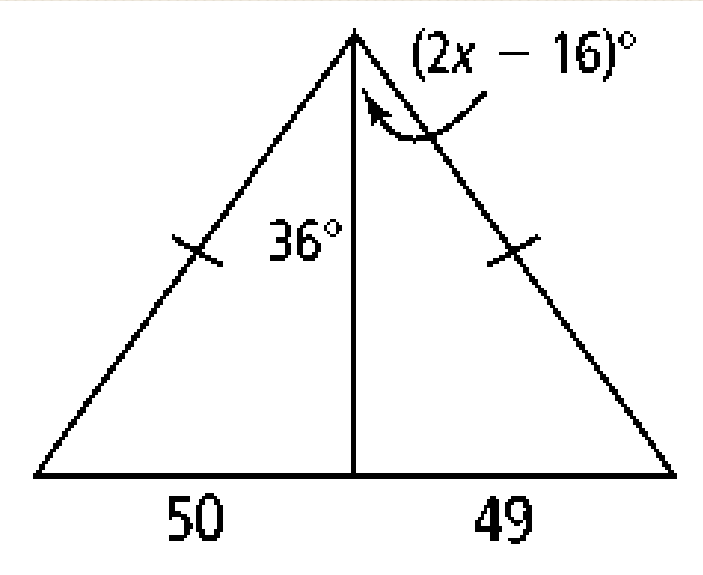
CLASS WORK

4. A crocodile opens his jaws at a 30° angle. He closes his jaws, then opens them again at a 36° angle. In which case is the distance between the tip of his upper jaw and the tip of his lower jaw greater? Explain.

The second time because of the Hinge Theorem

EXIT PROBLEMS

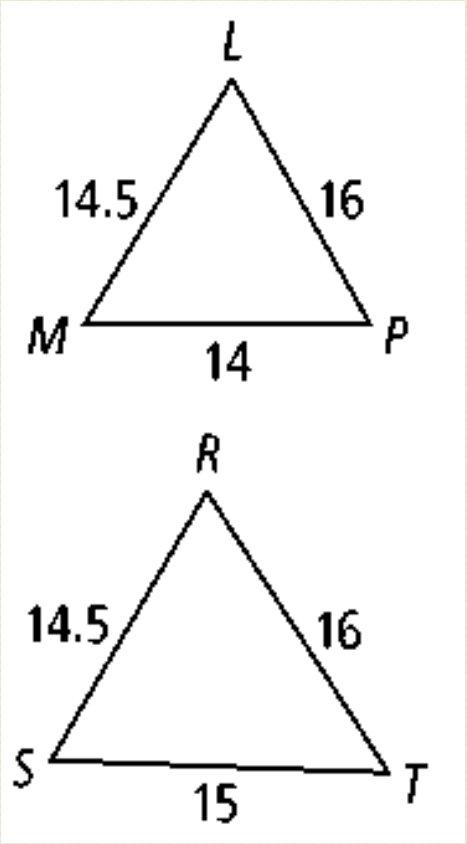
7. Find the range of values for x.



$2x - 16 < 36$
$2x < 52$
$x < 26$

$2x - 16 > 0$
$2x > 16$
$x > 8$
$8 < x < 26$

8. Write an inequality for angles L and R.



$m\angle R > m\angle L$

SUMMARY

- In an indirect proof, you first assume temporarily the opposite of what you want to prove. Then you show that this temporary assumption leads to a contradiction, so the prove statement must be true.
- In a triangle, the sum of any two side lengths is greater than the third side.
- If two sides are not congruent, then the larger angle lies opposite the longer side.
- If two angles are not congruent, then the longer side lies opposite the larger angle.

SUMMARY

The Hinge Theorem states that if two sides of one triangle are congruent t o two sides of another triangle, and the included angles are not congruent, then the longer third side is opposite the larger included angle.

LEARNING RUBRIC

- Got It: Proves Theorems with indirect proofs
- Almost There: Orders sides and angles of a triangle by size
- Moving Forward: Finds the range of possible third sides for a triangle
- Getting Started: Determines if sides of a triangle are possible

HOMEWORK

- 5-5: Pages 348 – 349

  20, 22, 24, 26, 30, 42, 48, 54

- 5-6: Pages 355 – 357

  10, 12, 14, 20, 26, 30, 32