5-1

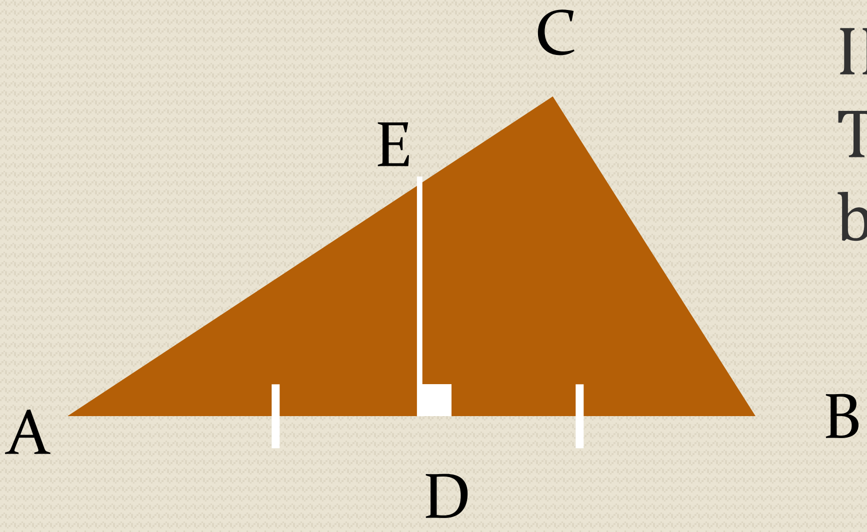
PERPENDICULAR AND ANGLE BISECTORS

- To use properties of perpendicular bisectors and angle bisectors
- To prove and apply theorems about perpendicular bisectors and angle bisectors

OBJECTIVES

KEY CONCEPTS

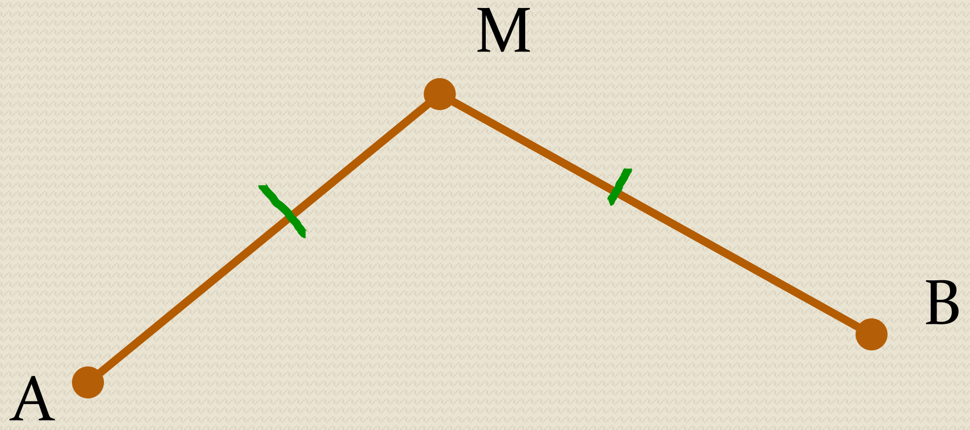
Perpendicular bisector of a triangle – is a segment that bisects a side and is perpendicular to that side



IF $AD = DB$, and $\overline{ED} \perp \overline{AB}$, Then $\overline{ED}$ is a perpendicular bisector of ΔABC

VOCABULARY

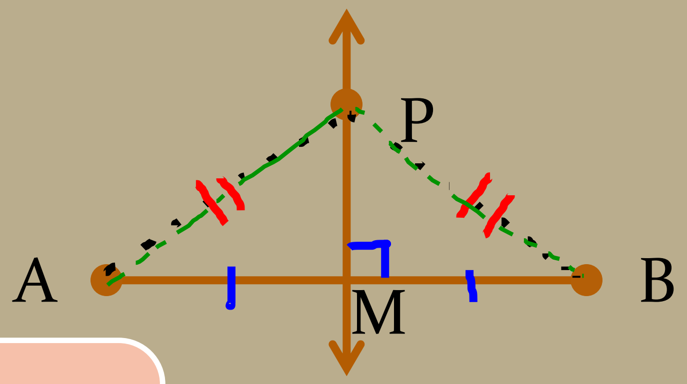
Equidistant – A point is equidistant from two objects if it is the same distance from the objects.



IF AM = MB, then point M is equidistant from points A and B.

KEY CONCEPT

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.



Perpendicular Bisector Theorem

If P lies on the perpendicular bisector of $\overline{AB}$, ($\overleftrightarrow{PM} \perp \overline{AB}$ and $\overline{MA} \cong \overline{MB}$), then $\overline{PA} \cong \overline{PB}$.

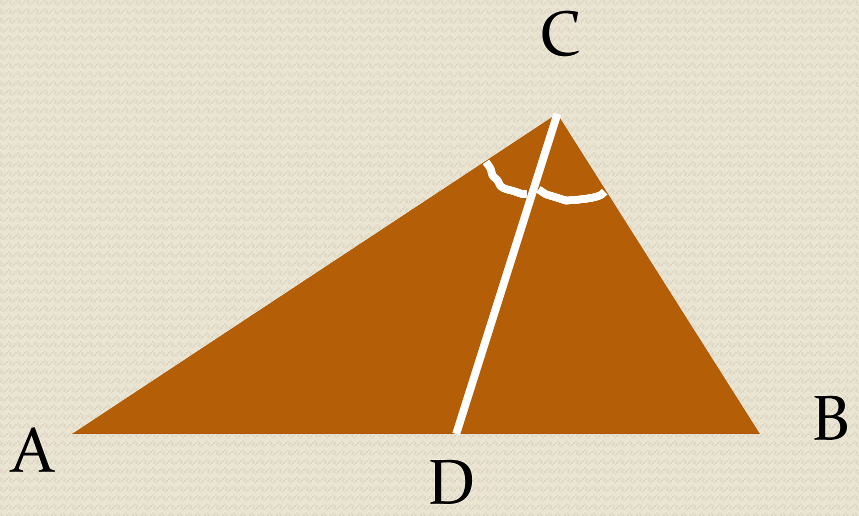
CONVERSE:

If $\overline{PA} \cong \overline{PB}$,

then P lies on the perpendicular bisector of $\overline{AB}$.

VOCABULARY

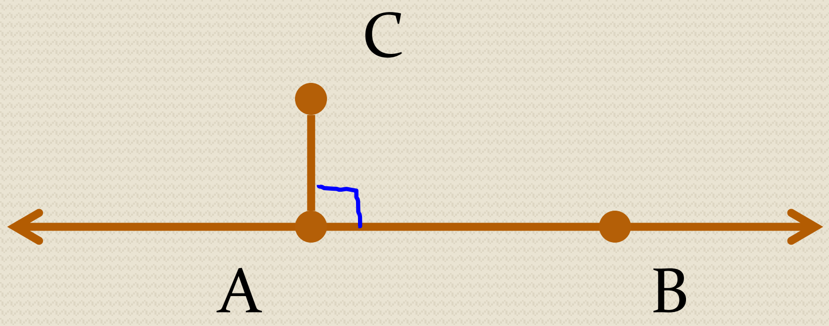
Angle bisector of a triangle – a segment that bisects an angle of a triangle



IF $\angle ACD \cong \angle DCB$, then $\overline{CD}$ is an angle bisector of ΔABC

VOCABULARY

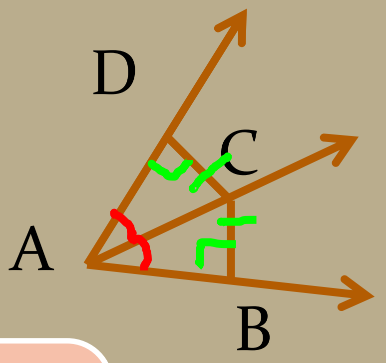
Distance from a point to a line – the length of the perpendicular segment from the point to the line. This is the shortest segment from the point to the line.



IF $\overline{CA} \perp \overleftrightarrow{AB}$, then $\overline{CA}$ represents the distance from point C to $\overleftrightarrow{AB}$.

KEY CONCEPT

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.



Angle Bisector Theorem

If $\overrightarrow{AC}$ bisects $\angle DAB$,

$\overline{CD} \perp \overrightarrow{AD}$, and $\overline{CB} \perp \overrightarrow{AB}$,

Then $\overline{CD} \cong \overline{CB}$.

CONVERSE:

If $\overline{CD} \perp \overrightarrow{AD}$, $\overline{CB} \perp \overrightarrow{AB}$,

and $\overline{CD} \cong \overline{CB}$,

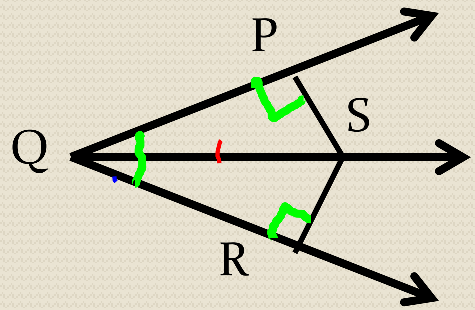
then $\overrightarrow{AC}$ bisects $\angle DAB$.

CLASS WORK

Prove the Angle Bisector Theorem

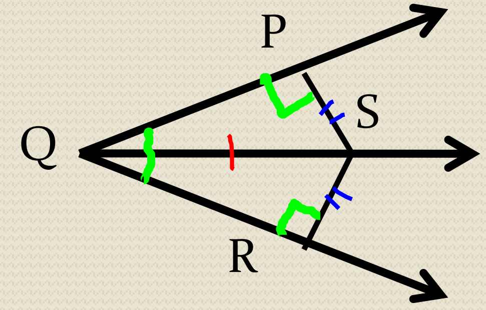
Given: $\overrightarrow{QS}$ bisects ∠PQR, $\overline{SP} \perp \overrightarrow{QP}$, $\overline{SR} \perp \overrightarrow{QR}$

Prove: SP = SR



Statements	Reasons
1) $\overrightarrow{QS}$ bisects ∠PQR, $\overline{SP} \perp \overrightarrow{QP}$, $\overline{SR} \perp \overrightarrow{QR}$	1) Given
2)	2)
3)	3)
4)	4)
5)	5)
6)	6)
7)	7)
8)	8)

CLASS WORK



Prove the Angle Bisector Theorem

Given:. $\overrightarrow{\boldsymbol{QS}}$ bisects ∠PQR, $\overline{SP} \perp \overrightarrow{QP}$, $\overline{SR} \perp \overrightarrow{QR}$

Prove: SP = SR

Statements	Reasons
1) $\overrightarrow{\boldsymbol{QS}}$ bisects ∠PQR, $\overline{SP} \perp \overrightarrow{QP}$, $\overline{SR} \perp \overrightarrow{QR}$	1) Given
2) ∠PQS ≅ ∠RQS	2) Definition of angle bisector.
3) ∠QPS and ∠QRS are right angles.	3) Definition of perpendicular.
4) ∠QPS ≅ ∠QRS	4) All right angles are congruent.
5) $\overline{QS} \cong \overline{QS}$	5) Reflexive property of ≅.
6) ΔQPS ≅ ΔQRS	6) AAS Theorem
7) $\overline{SP} \cong \overline{SR}$	7) CPCTC
8) SP = SR	8) Definition of congruent seg.

CLASS WORK

Use the figure below for Exercises 1–3.

1. What is the relationship between $\overline{LN}$ and $\overline{MO}$?

⊥ bisectors

2. What is the value of x?

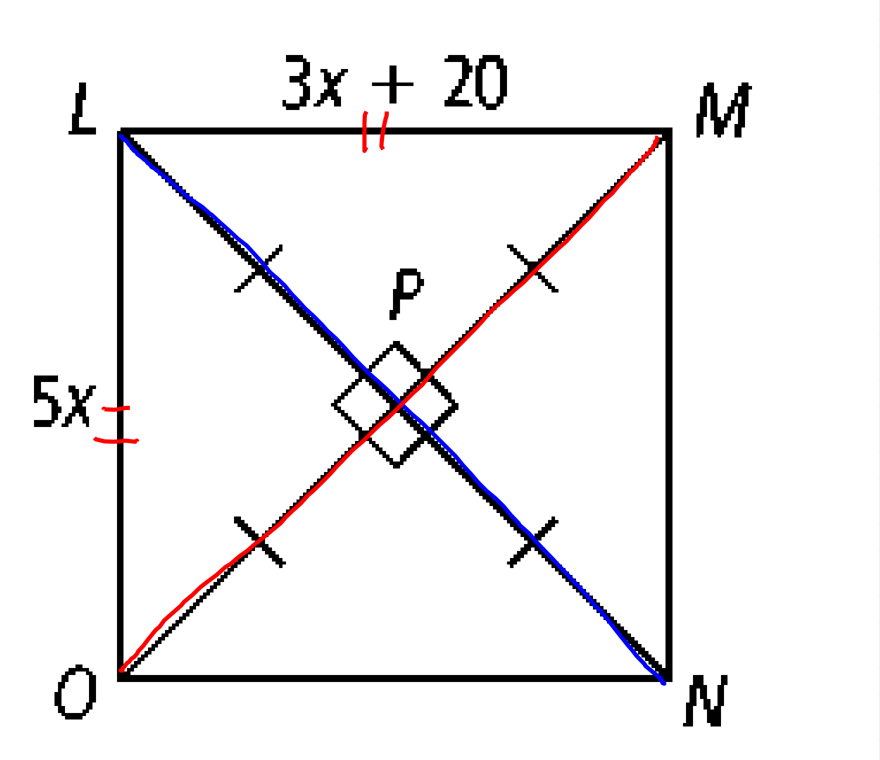
$5x = 3x + 20$

$2x = 20$

$x = 10$

3. Find LM.

$LM = LO = 5(10) = 50$



CLASS WORK

Use the figure below for Exercises 4 - 8.

4. According to the figure, how far is *A* from $\overline{CD}$? from $\overline{CB}$? 15

5. How is $\overrightarrow{CA}$ related to $\angle DCB$? Explain. ∠ bisector

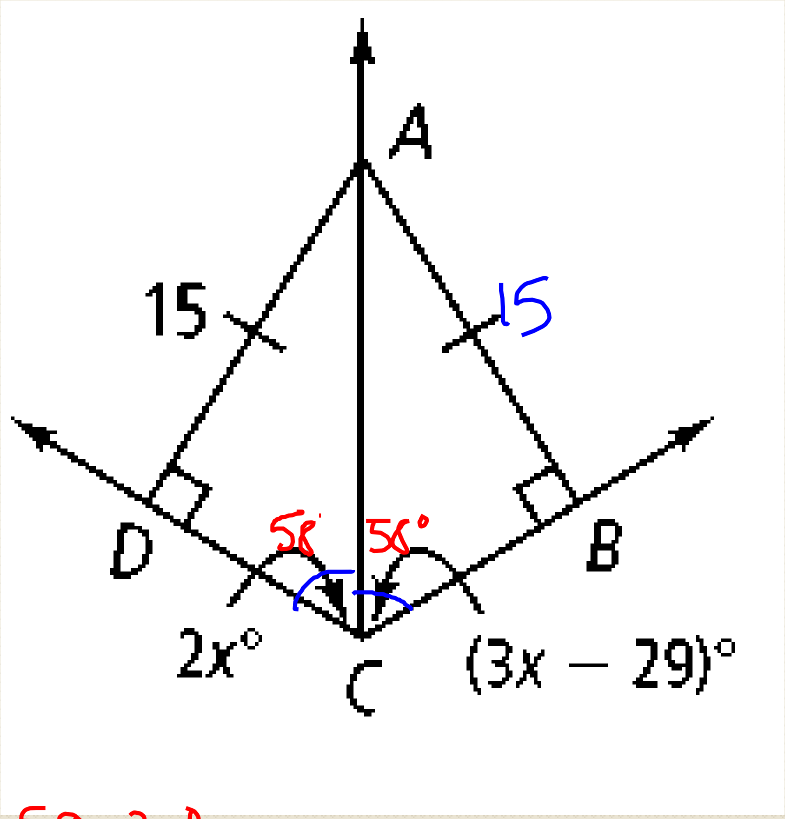
6. Find the value of *x*. $3x-29=2x$

$x=29$

7. Find $m\angle ACD$ and $m\angle ACB$. $m\angle ACD = m\angle ACB = 2(29) = 58°$

8. Find $m\angle DAC$ and $m\angle BAC$.

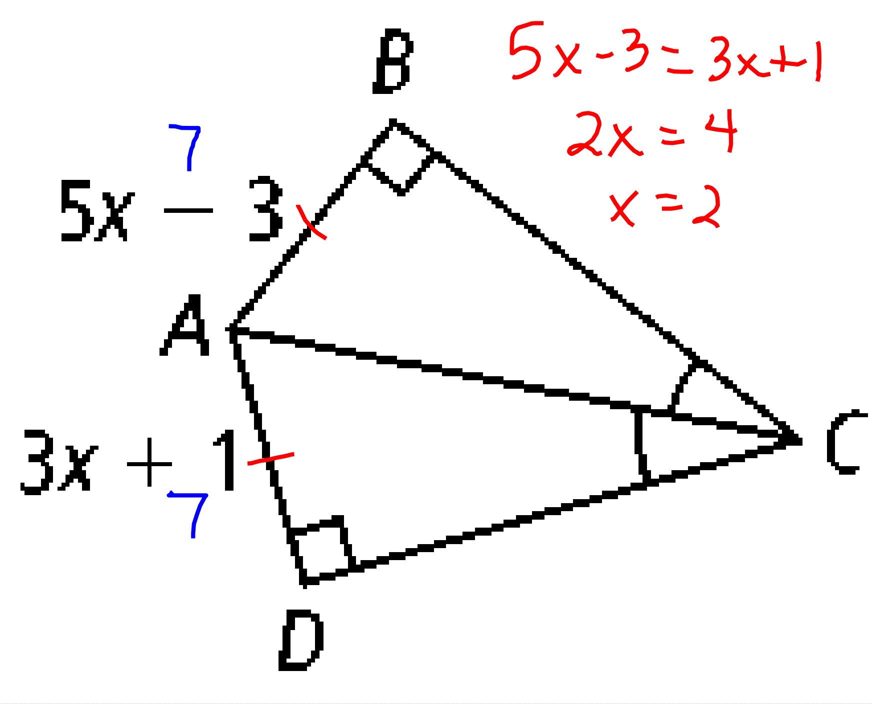
$m\angle DAC = m\angle BAC = 90-58 = 32°$



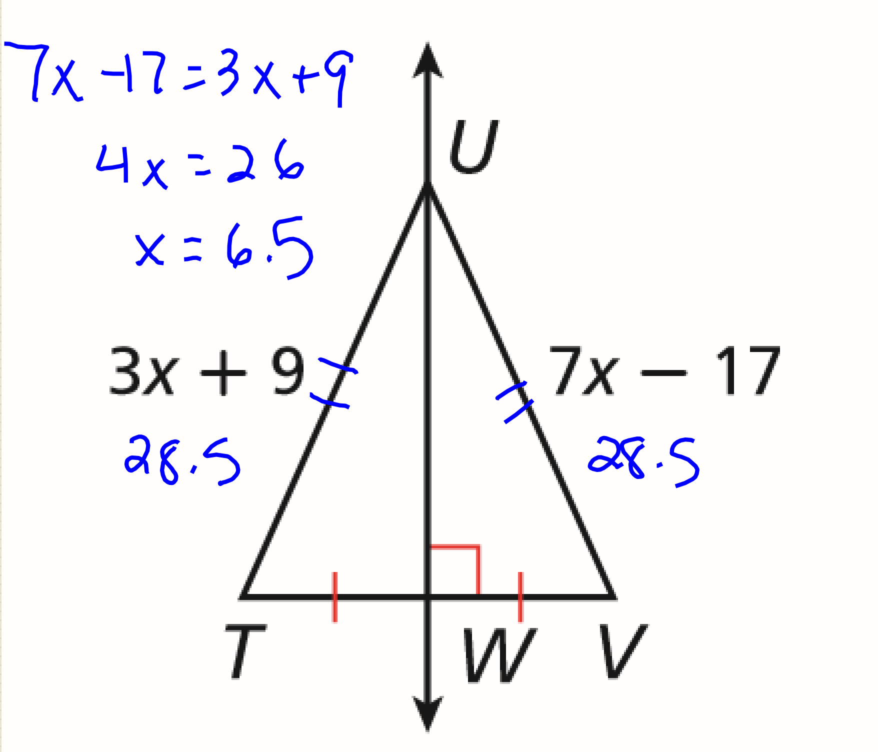
CLASS WORK

Find the indicated variables and measures.

9. x, BA, DA



10. x, TU, UV



EXIT PROBLEM

Write an equation in slope-intercept form for the perpendicular bisector of the segment with endpoints C(6,-5) and D(10,1)

Segment CD

$$m = \frac{1+5}{10-6} = \frac{6}{4} = \frac{3}{2}$$

$$\text{midpt}\left(\frac{6+10}{2}, \frac{-5+1}{2}\right) \rightarrow (8, -2)$$

x_1, y_1

$\perp$ bisector

$$m = -\frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y + 2 = -\frac{2}{3}(x - 8)$$

$$y + 2 = -\frac{2}{3}x + \frac{16}{3} - \frac{6}{3}$$

$$-2$$

$$y = -\frac{2}{3}x + \frac{10}{3}$$

1. The Perpendicular Bisector Theorem together with its converse states that P is equidistant from A and B if and only if P is on the perpendicular bisector of $\overline{AB}$.

2. The Angle Bisector Theorem together with its converse states that P is equidistant from the sides of an angle if and only if P is on the angle bisector.

SUMMARY

LEARNING RUBRIC

- Got It: Proves Perpendicular Bisector and Angle Bisector Theorems
- Almost There: Write equations for perpendicular bisectors of line segments
- Moving Forward: Applies the Perpendicular Bisector and the Angle Bisector Theorems to problem solving
- Getting Started: Identifies perpendicular bisectors and angle bisectors

HOMEWORK

Pages 316 – 318

12 – 28 even;

33, 36