4-3
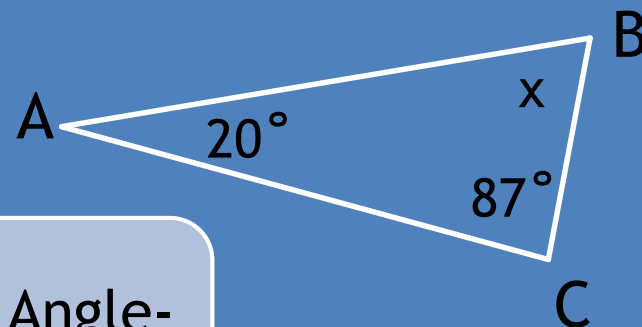# Angle Relationships in Triangles

OBJECTIVES

❖ To find the measures of interior and exterior angles of triangles

❖ To apply theorems about the interior and exterior angles of triangles

KEY CONCEPTS

The sum of the measures of the angles of a triangle is 180.

Triangle Angle-Sum Theorem
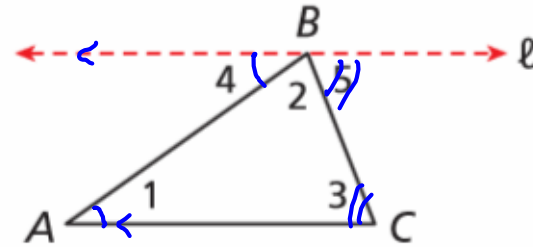
$m\angle A + m\angle B + m\angle C = 180$
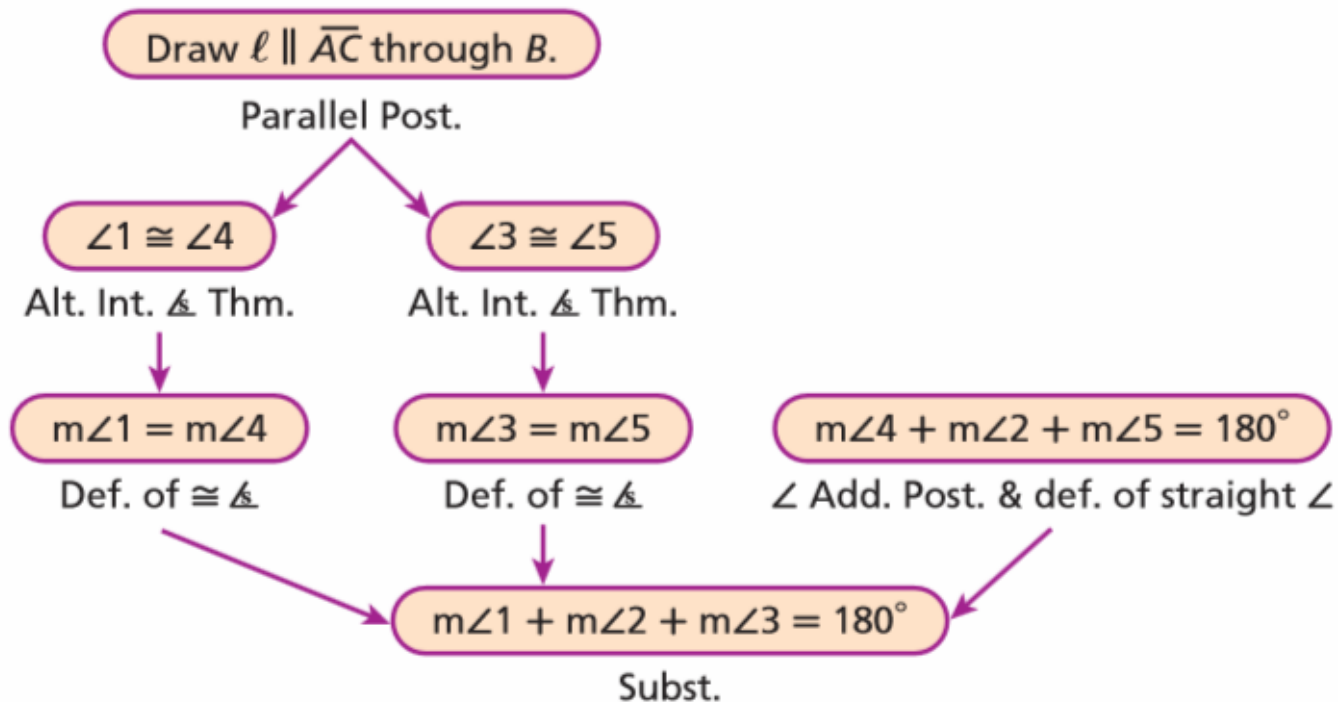
$20 + x + 87 = 180$

$x + 107 = 180$

$x = 73$

Given: $\triangle ABC$
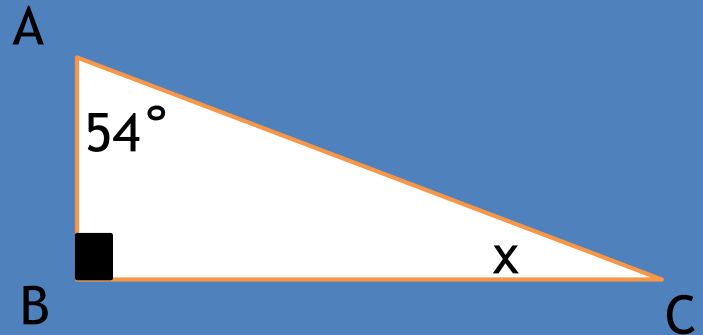**Prove:** $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Proof:

Draw $\ell \parallel \overline{AC}$ through B.

Parallel Post.

$\angle 1 \cong \angle 4$

Alt. Int. ∠ Thm.

$\angle 3 \cong \angle 5$

Alt. Int. ∠ Thm.

$m\angle 1 = m\angle 4$

Def. of $\cong$ ∠

$m\angle 3 = m\angle 5$

Def. of $\cong$ ∠

$m\angle 4 + m\angle 2 + m\angle 5 = 180°$

∠ Add. Post. & def. of straight ∠

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Subst.

Proof of Triangle Angle Sum Theorem

KEY CONCEPTS
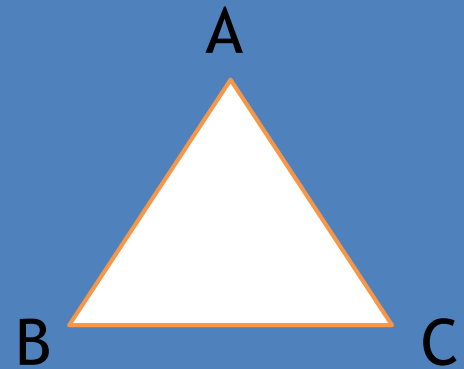
The acute angles of a right triangle are complementary.

A

54°

B

x

C

Corollary 4-3-2

$\angle A$ and $\angle C$ are complementary.

$$m\angle A + m\angle C = 90$$
$$54 + x = 90$$
$$x = 36$$

KEY CONCEPTS

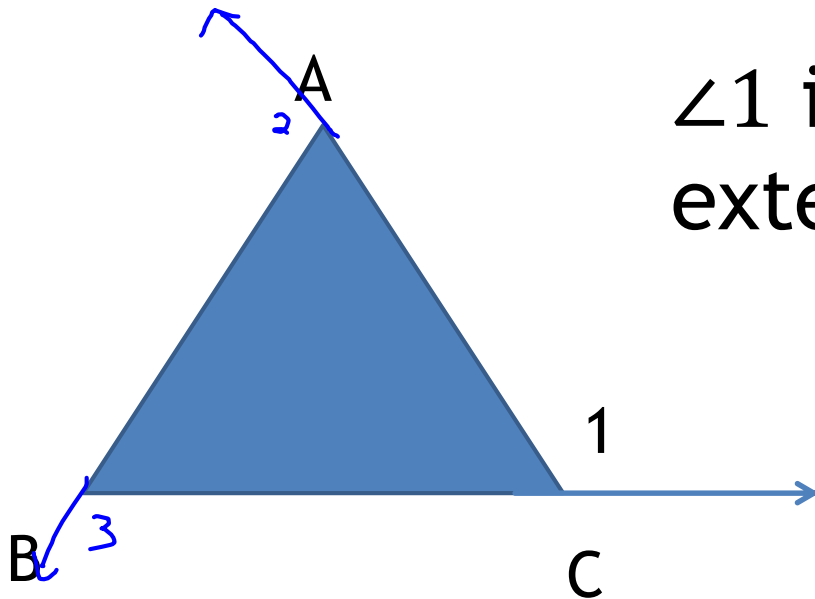The measure of each angle of an equiangular triangle is 60°.

Corollary 4-3-3

$\angle A \cong \angle B \cong \angle C$

$m\angle A = m\angle B = m\angle C = 60°$
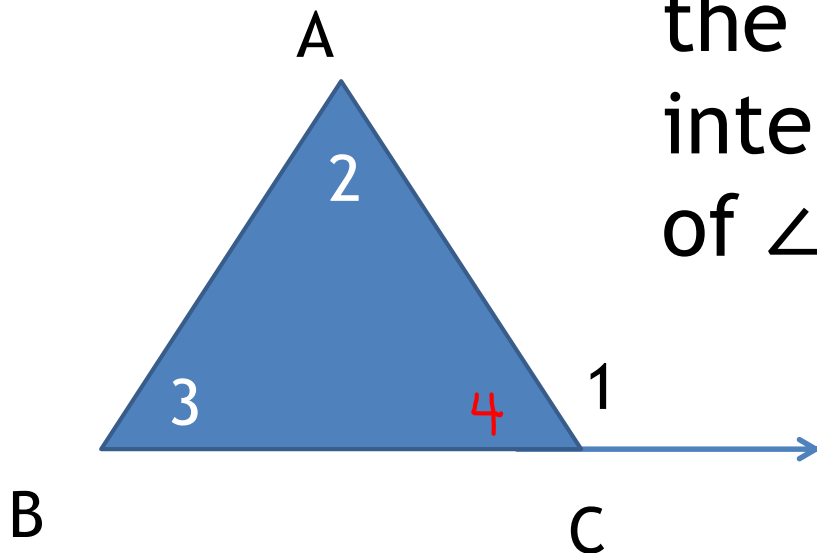
VOCABULARY

Exterior angle of a polygon – angle formed by a side and an extension of an adjacent side

$\angle 1$ is an exterior angle.

VOCABULARY

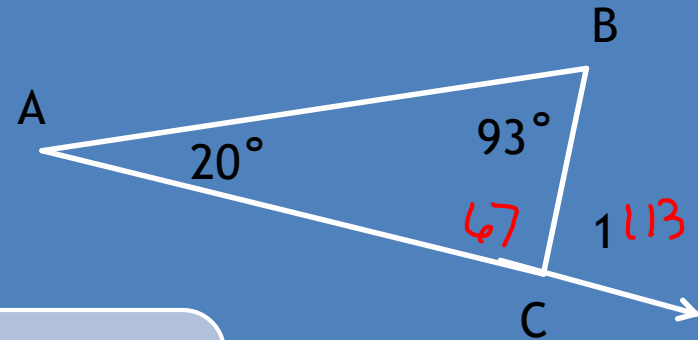Remote interior angles – the two nonadjacent interior angles of an exterior angle

∠2 and ∠3 are the remote interior angles of ∠1.

KEY CONCEPTS

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

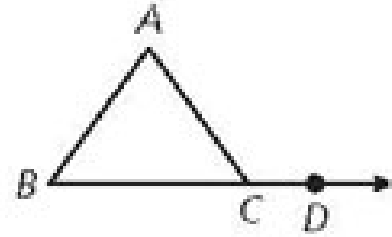Exterior Angle Theorem

$m\angle 1 = m\angle A + m\angle B$

$m\angle 1 = 20 + 93$
$m\angle 1 = 113$

Given: $\triangle ABC$ with exterior angle $\angle ACD$
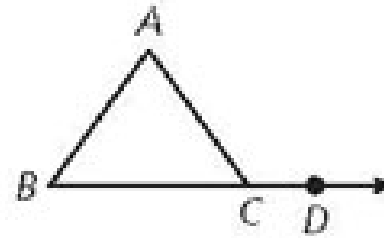
Prove: $m\angle ACD = m\angle A + m\angle B$

Statements	Reasons

PROOF OF EXTERIOR ANGLE THEOREM

Given: ΔABC with exterior angle $\angle ACD$

Prove: $m\angle ACD = m\angle A + m\angle B$
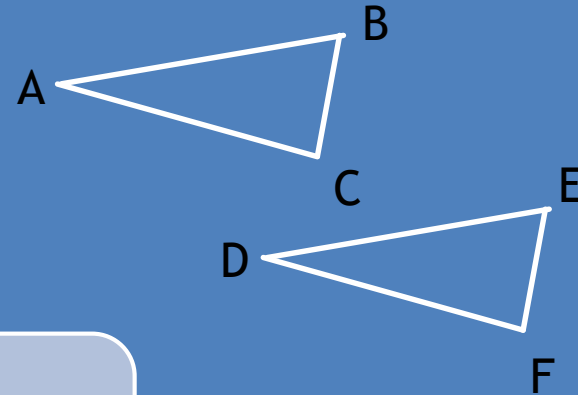
Statements	Reasons
ΔABC with exterior $\angle ACD$	Given
$m\angle A + m\angle B + m\angle ACB = 180$	$\Delta \angle$ Sum Thm
$m\angle ACB + m\angle ACD = 180$	Lin. Pair Thm.
$m\angle ACB + m\angle ACD = m\angle A + m\angle B + m\angle ACB$	Trans. Prop of $=$ (subst.)
$m\angle ACD = m\angle A + m\angle B$	Subtr. prop. of $=$

PROOF OF EXTERIOR ANGLE THEOREM

KEY CONCEPTS

If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.
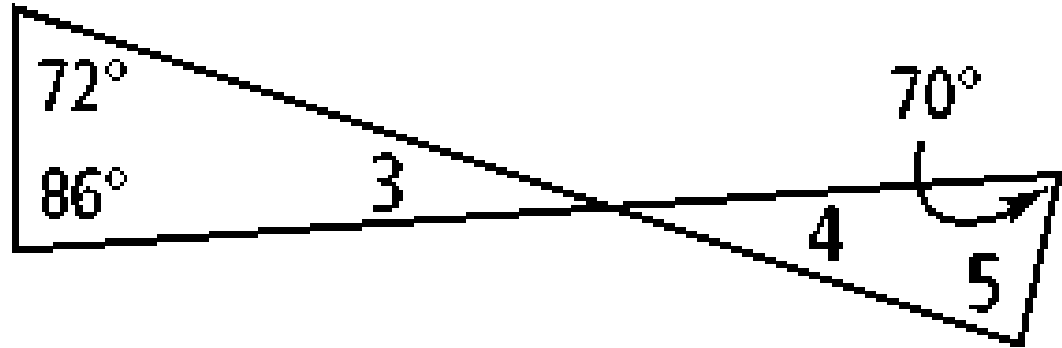

Third Angles Theorem

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$,

then $\angle C \cong \angle F$
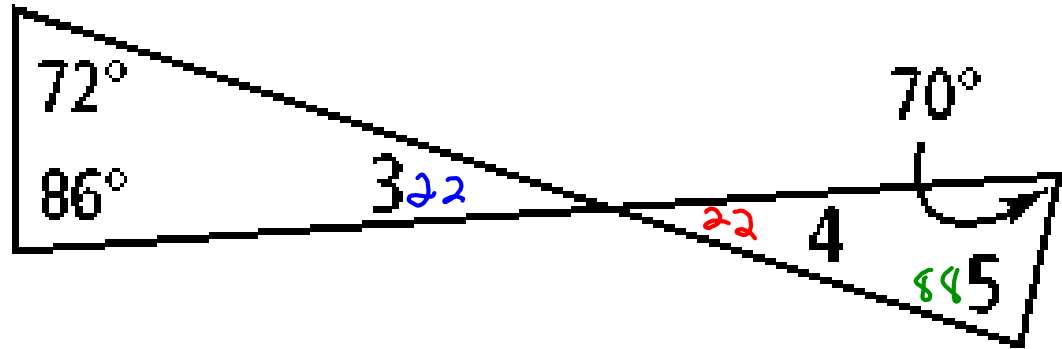
CLASS WORK

1. Find each missing angle measure.

CLASS WORK

1. Find each missing angle measure.

$m\angle3 + 72 + 86 = 180$

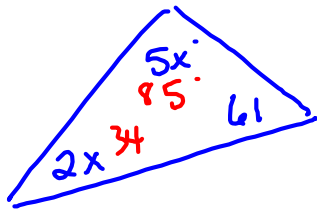$m\angle3 + 158 = 180$

$m\angle3 = 22°$

$m\angle4 = m\angle3 = 22°$

$m\angle5 = 180 - 70 - 22 = 88°$

CLASS WORK

2. The measure of one angle in a triangle is 61˚. The other two angles are in a ratio of 2 : 5. Find the measures of the angles.

CLASS WORK

2. The measure of one angle in a triangle is 61°. The other two angles are in a ratio of 2 : 5. Find the measures of the angles.
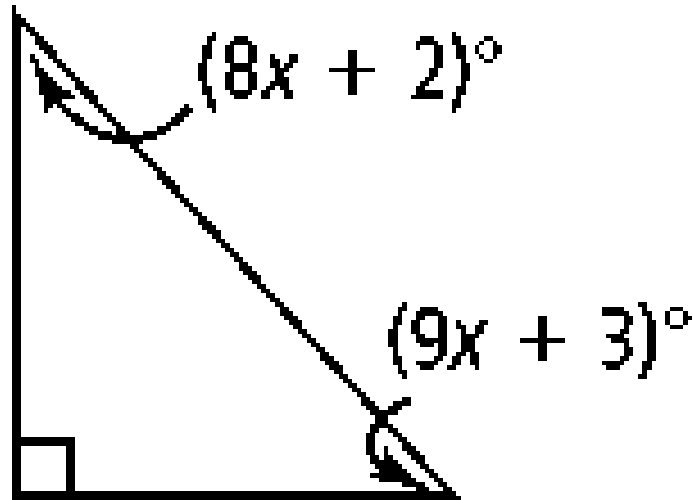
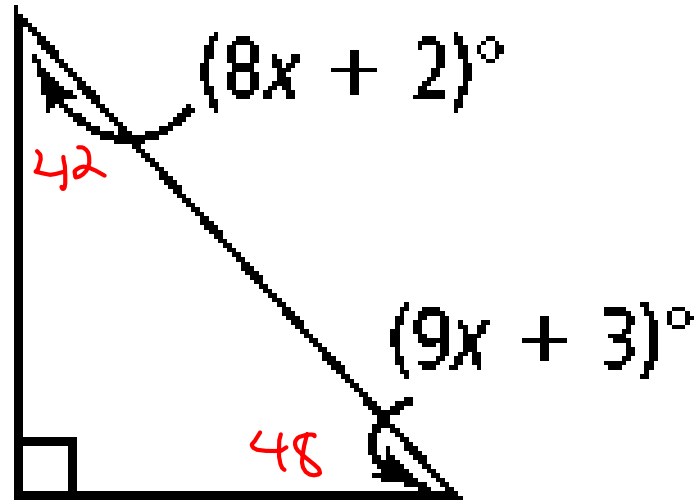$$2x + 5x + 61 = 180$$
$$7x + 61 = 180$$
$$7x = 119$$
$$x = 17$$

CLASS WORK

3. Find the value of x and the measures of the acute angles.

$(8x + 2)°$

$(9x + 3)°$

CLASS WORK

3. Find the value of x and the measures of the acute angles.

$(8x + 2)°$

42

$(9x + 3)°$

48

$$90 + 8x + 2 + 9x + 3 = 180$$
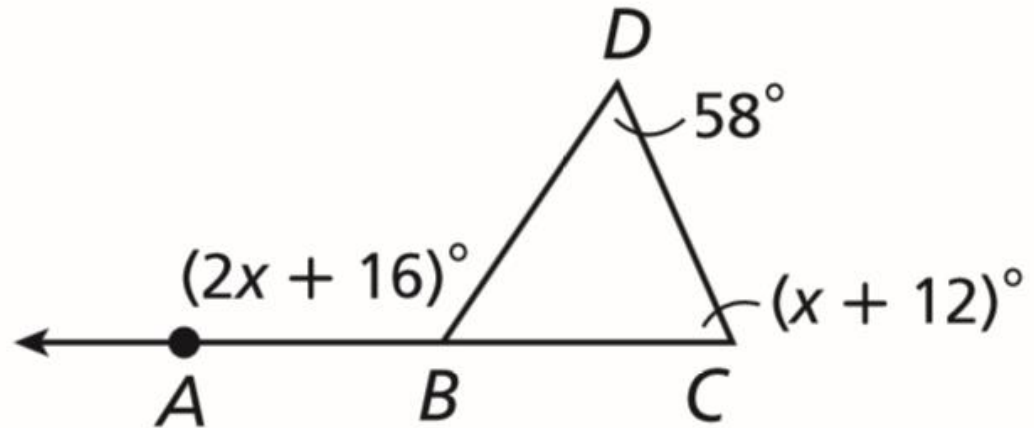$$-90 \qquad\qquad -90$$

$$8x + 2 + 9x + 3 = 90$$

$$17x + 5 = 90$$

$$17x = 85$$

$$x = 5$$

CLASS WORK

4. Find $m\angle ABD$.

CLASS WORK

4. Find $m\angle ABD$.

124

$108 + 16$

$2(54) + 16$
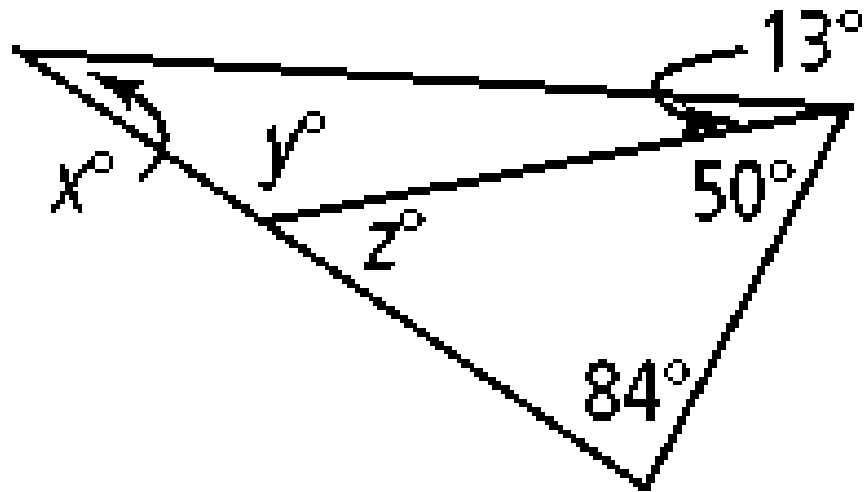
$124 = (2x + 16)°$

D

58°

$(x + 12)°$

A   B   C
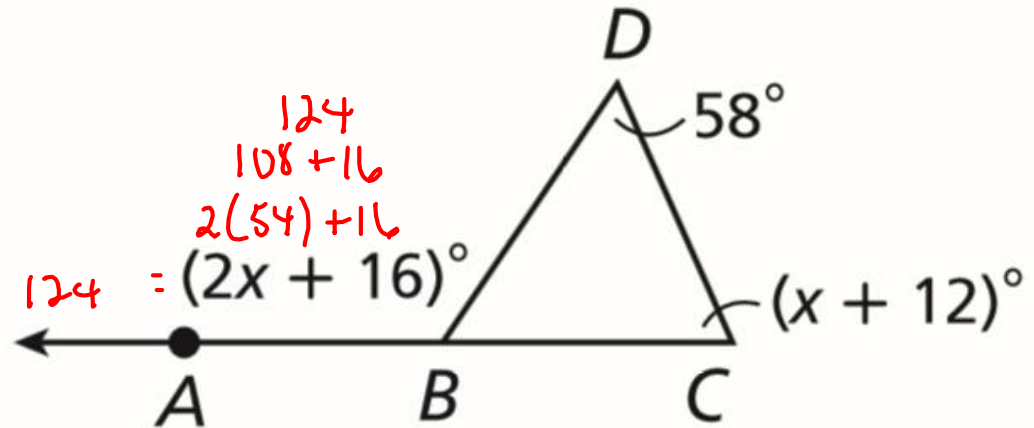
$2x + 16 = 58 + x + 12$
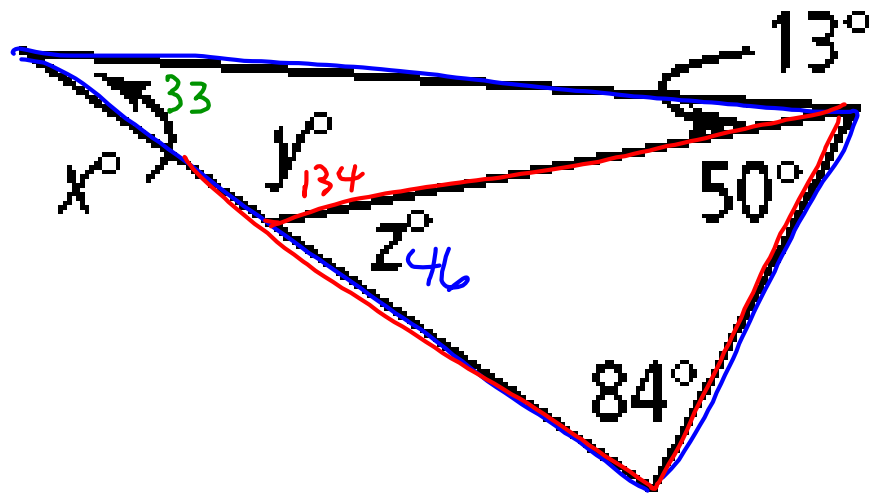
$2x + 16 = x + 70$

$x = 54$

CHALLENGE

6. Find the value of each variable.

CHALLENGE

6. Find the value of each variable.

$$Z = 180 - 84 - 50 = 46$$

$$y = 180 - 46 = 134$$

$$X = 180 - 13 - 134 = 33$$

LEARNING RUBRIC

▶ Got It: Applies concepts to prove congruence and find angle measures in complex/real world situations

▶ Almost There: Represents and applies concepts to solve for angle measures

▶ Moving Forward: Solves for interior and exterior angle measures in more complex situations that are represented

▶ Getting Started: Solves for interior and exterior angles in simple, represented settings

❖   The sum of the measures of a triangle is 180˚.

❖   The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

SUMMARY

HOMEWORK

Pages 236 – 238:

16 – 26 even;

30 – 36 even;

38, 40