Congruence and Transformations

4-1

1. Draw, identify, and describe transformations in the coordinate plane

2. Use properties of rigid motions to determine whether figures are congruent and to prove figures congruent.

OBJECTIVES

REVIEW AND EXTEND

Representing Transformations in the Coordinate Plane

TRANSFORMATION	COORDINATE MAPPING AND DESCRIPTION
Translation	$(x, y) \rightarrow (x + a, y + b)$ Translation a units horizontally and b units vertically
Reflection	$(x, y) \rightarrow (-x, y)$ Reflection across y-axis $(x, y) \rightarrow (x, -y)$ Reflection across x-axis
Rotation	$(x, y) \rightarrow (y, -x)$ Rotation about (0, 0), 90° clockwise $(x, y) \rightarrow (-y, x)$ Rotation about (0, 0), 90° counterclockwise $(x, y) \rightarrow (-x, -y)$ Rotation about (0, 0), 180°
Dilation	$(x, y) \rightarrow (kx, ky), k > 0$ Dilation with scale factor k and center (0, 0)

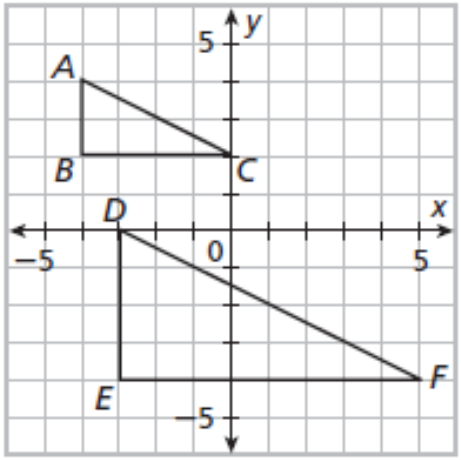
Transformations and Congruence

Translations, reflections, and rotations produce images that are congruent to their preimages.

Dilations with scale factor $k \neq 1$ produce images that are not congruent to their preimages.

EXAMPLES

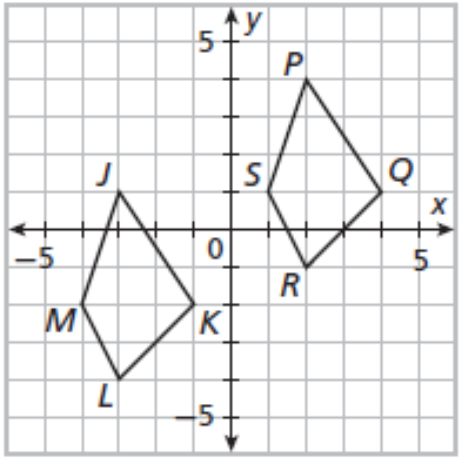
Use the definition of congruence in terms of rigid motions to determine whether the two figures are congruent and explain your answer.



A $\triangle ABC$ and $\triangle DEF$ have different sizes.

Since rigid motions preserve distance, there is no sequence of rigid motions that will map $\triangle ABC$ to $\triangle DEF$.

Therefore, $\triangle ABC \ncong \triangle DEF$



B You can map $JKLM$ to $PQRS$ by the translation that has the following coordinate notation:

$(x, y) \rightarrow (x+5, y+3)$

A translation is a rigid motion.

Therefore, $\triangle JKLM \cong \triangle PQRS$

EXAMPLES

For each pair of congruent figures, find a sequence of rigid motions that maps one figure to the other.

A You can map $\triangle ABC$ to $\triangle RST$ by a reflection followed by a translation. Provide the coordinate notation for each.

y-axis

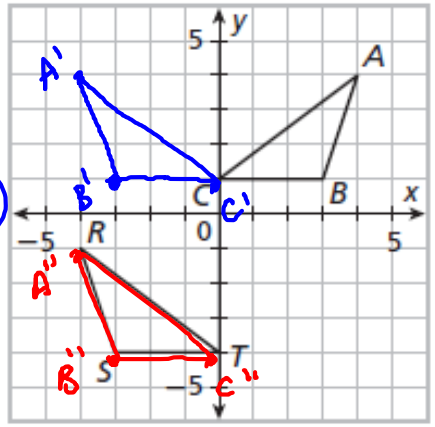
Reflection: $(x, y) \rightarrow (-x, y)$ $A(4,4) \rightarrow A'(-4,4)$

$A''(-4,-1)$

Followed by...

Translation: $(x, y) \rightarrow (x, y-5)$

$\triangle ABC \cong \triangle RST$



B You can map $\triangle DFG$ to $\triangle HJK$ by a rotation followed by a translation. Provide the coordinate notation for each.

90°CC

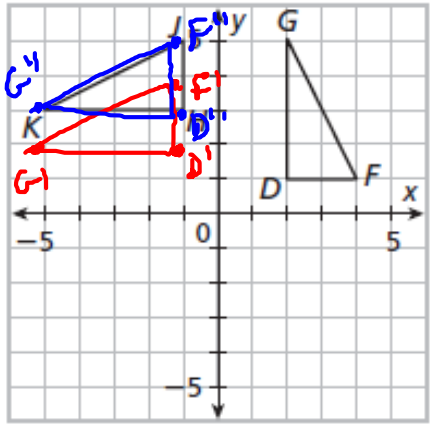
Rotation: $(x, y) \rightarrow (-y, x)$ $D(2,1) \rightarrow D'(-1,2)$

$D''(-1,3)$

Followed by...

Translation: $(x, y) \rightarrow (x, y+1)$

$\triangle DFG \cong \triangle HJK$



EXAMPLES

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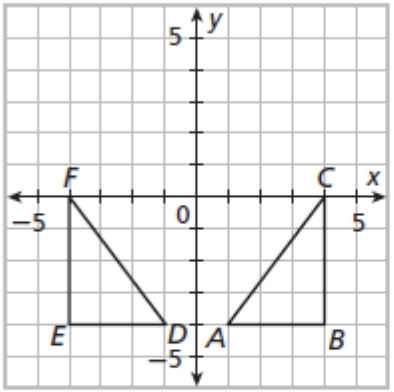
1.



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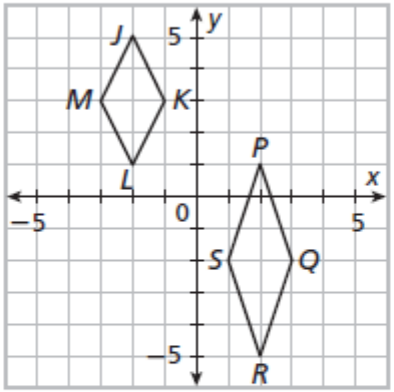

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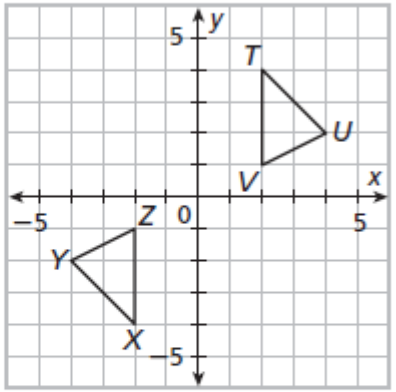

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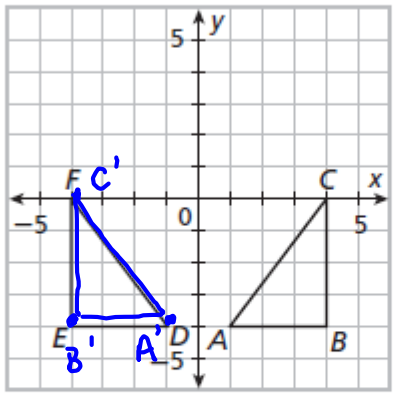


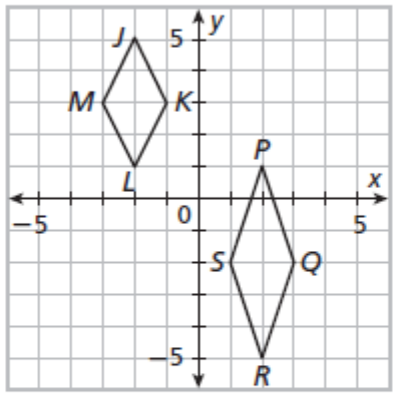
refl. across y-axis

$(x,y) \to (-x,y)$

$A(1,-4) \to A'(-1,-4)$

$\triangle ABC \cong \triangle DEF$

2.

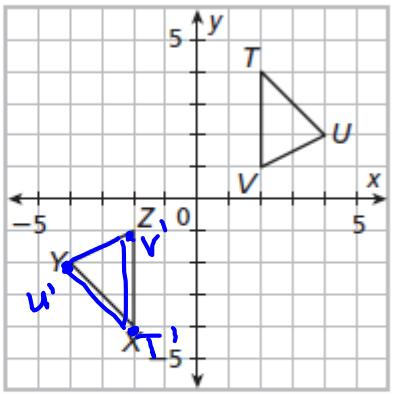


diff sizes;

no series of rigid motions to map one onto the other

$JKLM \ncong PQRS$

3.



180° rotation

$(x,y) \to (-x,-y)$

$T(2,4) \xrightarrow{T} (-2,-4)$

$\triangle TUV \cong \triangle XYZ$

EXAMPLES

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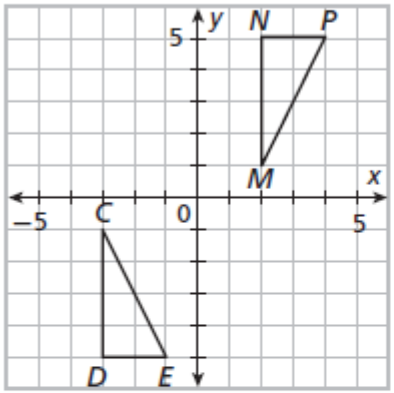
4.



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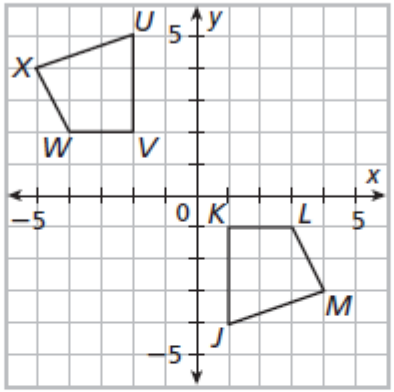

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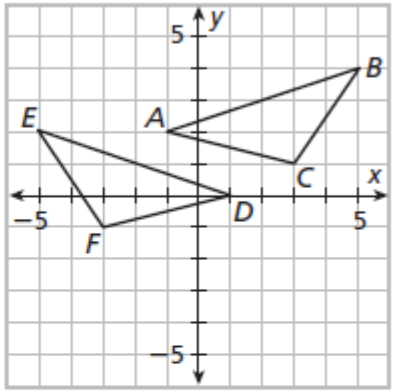

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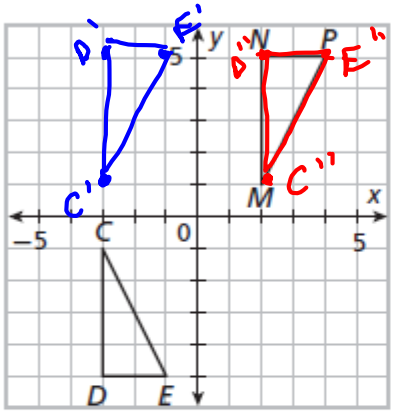


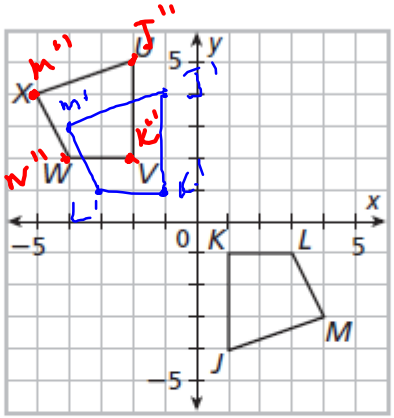
refl (x-axis) → trans.

$(x,y) \to (x,-y) \to (x+5,y)$

$C(-3,-1) \to C'(-3,1) \to C''(2,1)$

$\triangle CDE \cong \triangle MNP$

5.



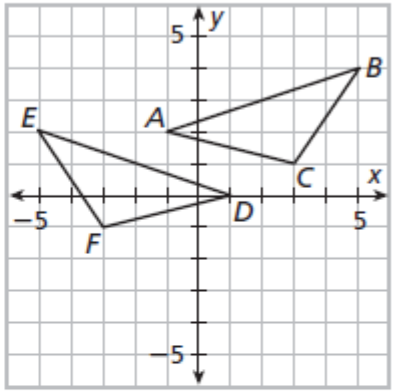
180° rotation → trans.

$(x,y) \to (-x,-y) \to (x-1,y+1)$

$K(1,-1) \to K'(-1,1) \to K''(-2,2)$

$JKLM \cong UVWX$

6.



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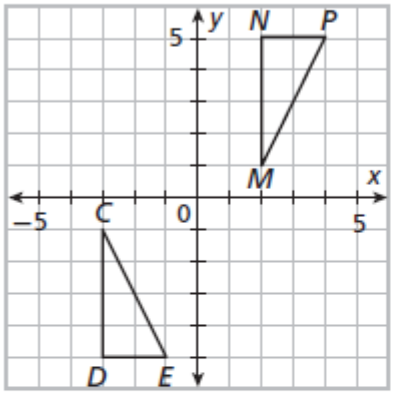

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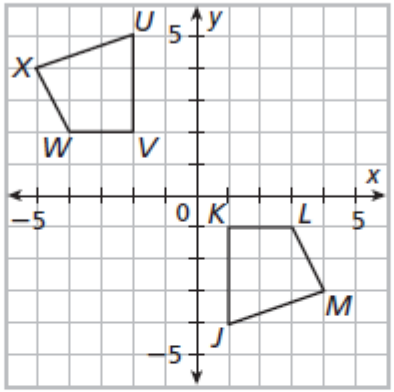

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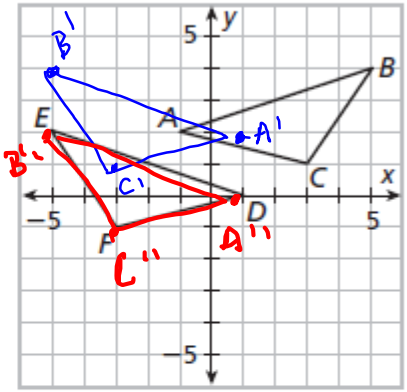


refl./y-axis → trans. ↓2

$(x,y) \to (-x,y) \to (x,y-2)$

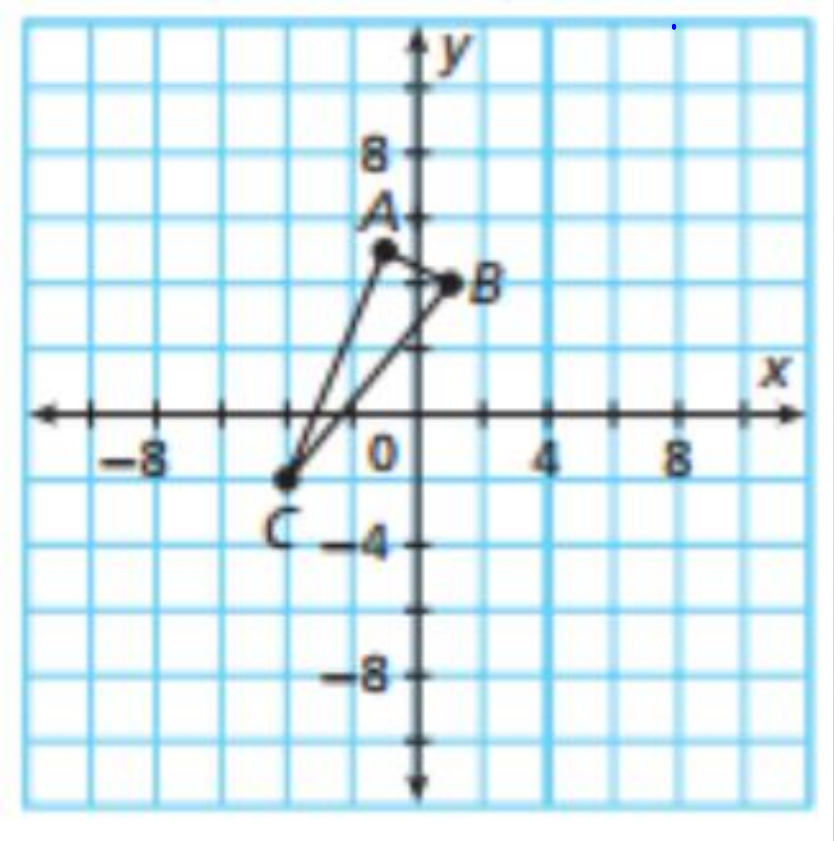
$A(-1,2) \to A'(1,2) \to A''(1,0)$

$\triangle ABC \cong DEF$

CHALLENGE

Apply the transformations M to the polygon with the given vertices. Name the coordinates of the image point. Identify and describe the transformations.

M: $(x, y) \rightarrow (x, -y) \rightarrow (x + 3, y)$



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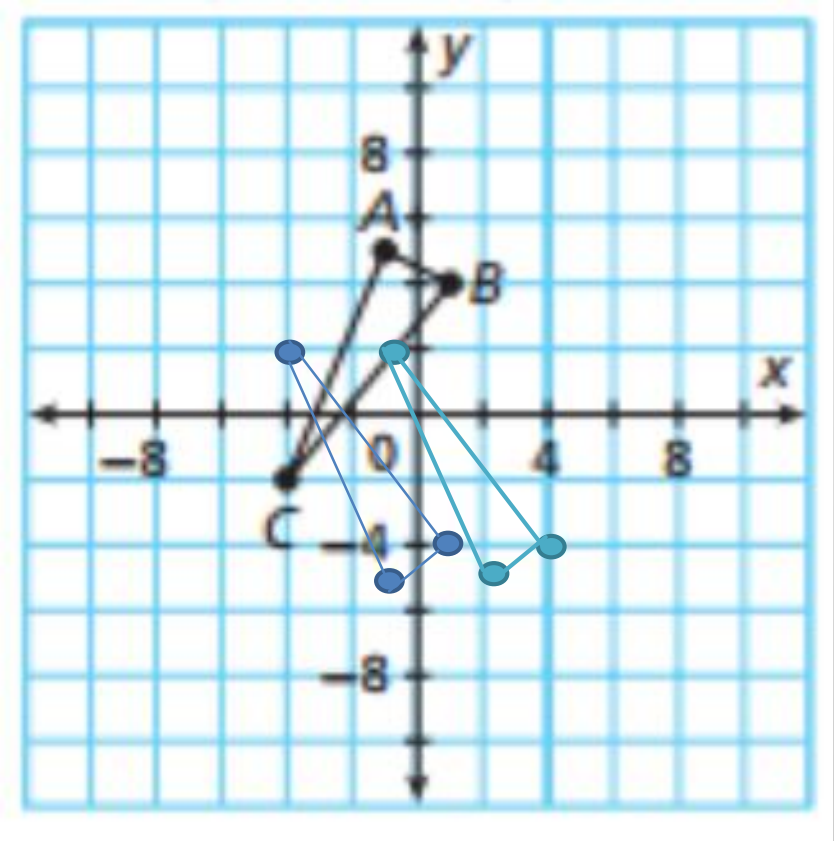

$A(-1,5) \rightarrow A'(-1,-5) \rightarrow A''(2,-5)$

$B(1,4) \rightarrow B'(1,-4) \rightarrow B''(4,-4)$

$C(-4,-2) \rightarrow C'(-4,2) \rightarrow C''(-1,2)$

Reflection across x-axis, then translation 3 units right

Translations, reflections, rotations, and combinations thereof produce images that are congruent to their preimages.

SUMMARY

LEARNING RUBRIC

- Got It: Identify transformation combinations and give multiple coordinate mapping steps to map a preimage onto its image to determine congruence
- Almost There: Identify a transformation and its coordinate mapping to map a preimage onto its image to determine congruence
- Moving Forward: Identify a transformation and show its coordinate mapping
- Getting Started: Graph and identify a transformation with given coordinate mapping

Pages 220 – 223:
14 – 26 even;
30, 36, 37

HOMEWORK