2-1

Inductive Reasoning

OBJECTIVES

1. To use inductive reasoning to identify patterns and make conjectures
2. To find counterexamples to disprove conjectures

VOCABULARY

Inductive Reasoning – the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern

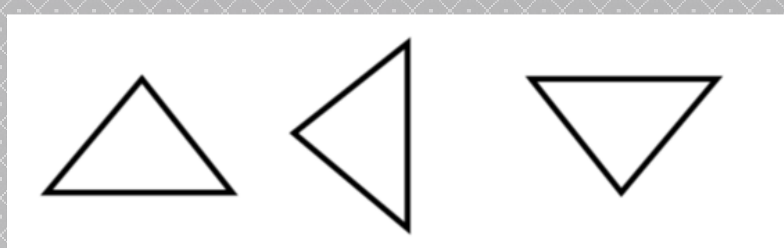
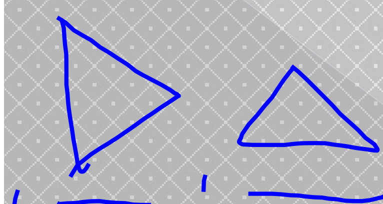
Example: January, March, May, July, Sept.

CLASS WORK

What is the pattern? What are the next two terms?

1. 7, 14, 21, 28… 35, 42

+7

2.  

rotate 90°

VOCABULARY

Conjecture – a statement you believe to be true based on inductive reasoning

- You have to test a conjecture more than once to use it.
- No amount of tests or examples proves a conjecture.

CLASS WORK

3. What conjecture can you make about the 25th term?

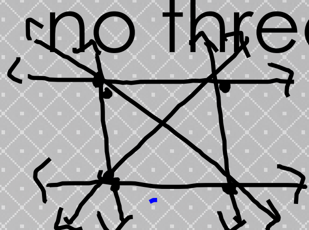
   1, 2, 3, 1, 2, 3, ... 1

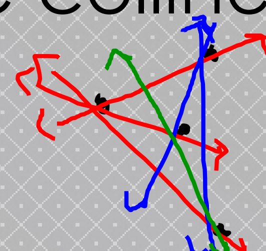
   24th 25th

4. The sum of two even numbers is even.

   8+4=12 ; 4+4=8 ; 2+2=4 ; 1232+200=1432

5. The number of lines formed by 4 points, no three of which are collinear is 6.





CLASS WORK

6. What conjecture can you make about the sum of the first 40 odd numbers?

First, gather information:

n	number		sum
1	1	1	1
2	3	1+3	4
3	5	1+3+5	9
4	7	1+3+5+7	16
5	9	16+9	25

(1, 4, 9, 16, ...)

$n^2 = 40^2 = 1600$

VOCABULARY

Counterexample – an example in which the conjecture is not true

- One counterexample disproves a conjecture
- A counterexample can be a drawing, a statement, or a number

CLASS WORK

7. Find a counterexample:

a) If an animal is green, then it is a frog. Snake

b) For every integer n, n^3 is positive. -2

c) Two complementary angles are not congruent. $45°$ and $45°$

SUMMARY

- The conclusions you reach based on inductive reasoning are conjectures
- You have to test a conjecture more than once before you use it.
- No amount of tests proves a conjecture.
- One counterexample disproves a conjecture.
- Postulates are formed using inductive reasoning (can't be proven or disproven).

LEARNING RUBRIC

- Got It: Uses inductive reasoning to solve Complex/real-world problems (make tables)
- Almost There: Use rules to “skip ahead”
- Moving Forward: Find counterexamples to disprove a conjecture
- Getting Started: Makes a conjecture based on observed patterns, and identify the next term

2-2

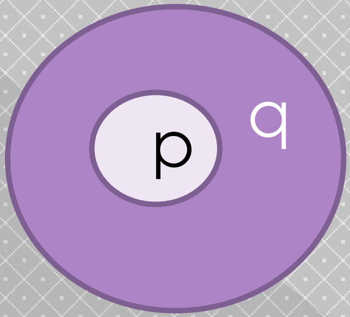
CONDITIONAL STATEMENTS

1. To identify and write conditional statements
2. To analyze the truth value of conditional statements
3. To write the inverse, converse, and contrapositive of a conditional statement

OBJECTIVES

VOCABULARY

Conditional statement – an if-then statement: p→q



Hypothesis – the part "*p*" following the if

Conclusion – the part "*q*" following the then

- The study of *if-then* statements and their truth values is foundation of reasoning.

CLASS WORK

Identify the hypothesis and conclusion of each conditional.

1. If today is Thanksgiving Day, then today is Thursday.
2. If a triangle has three congruent sides, then it is equilateral.
3. A number is a rational number if it is an integer.

CLASS WORK

Write a conditional statement from each of the following.

4. An obtuse triangle has exactly one obtuse angle. If a Δ is obtuse, then it has exactly one obtuse $\angle$.

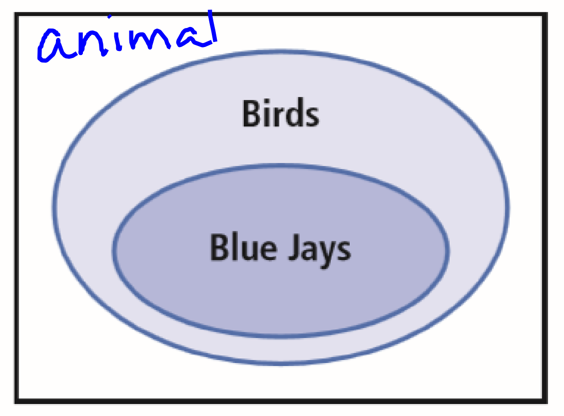
5. A catfish is a fish that has no scales.

If a fish is a catfish, then it has no scales.

CLASS WORK

Write the conditional statement that the Venn diagram illustrates.

6.



If an animal is a bluejay, then it is a bird.

VOCABULARY

Truth value – A conditional is either “true” or “false”

CLASS WORK

Determine if the conditional is true or false? If it is false, find counterexample.

7. If this month is September, then the next month is October. T

8. If two angles are acute, then they are congruent. F 20° and 70°

VOCABULARY

Negation – opposite of a statement

- the opposite of “p” is written as “~p”

KEY CONCEPTS

Statement	How to Write it	Symbols	How to Read it	Example
Conditional	Use the given hypothesis and conclusion.	$p \rightarrow q$	"if p, then q"	If two angles are complementary, then their measures sum to 90.
Converse				
Inverse				
Contrapositive				

KEY CONCEPTS

Statement	How to Write it	Symbols	How to Read it	Example
Conditional	Use the given hypothesis and conclusion.	$p \rightarrow q$	"if p, then q"	If two angles are complementary, then their measures sum to 90.
Converse	Exchange the hypothesis and conclusion.	$q \rightarrow p$	"if q, then p"	If the sum of two angles is 90,then the angles are complementary.
Inverse				
Contrapositive				

KEY CONCEPTS

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Inverse	Negate both the hypothesis and the conclusion.	$\sim p \rightarrow \sim q$	"if not p, then not q"	If two angles are not complementary, then their measures do not sum to 90.
Contrapositive				

KEY CONCEPTS

Statement	How to Write it	Symbols	How to Read it	Example
Conditional	Use the given hypothesis and conclusion.	$p \rightarrow q$	"if p, then q"	If two angles are complementary, then their measures sum to 90.
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Contrapositive	Exchange **AND** negate hypothesis and conclusion	$\sim q \rightarrow \sim p$	"if not q, then not p"	If the sum of two angles is not 90, then the angles are not complementary.

KEY CONCEPTS

Sometimes, all four statements have the same truth value. The conditional and the contrapositive will always have the same truth value. They are logically equivalent.

Statement	How to Write it	Symbols	How to Read it	Example
Conditional	Use the given hypothesis and conclusion.	$p \rightarrow q$	"if p, then q"	If two angles are complementary, then their measures sum to 90.
Converse	Exchange the hypothesis and conclusion.	$q \rightarrow p$	"if q, then p"	If the sum of two angles is 90,then the angles are complementary.
Inverse	Negate both the hypothesis and the conclusion.	$\sim p \rightarrow \sim q$	"if not p, then not q"	If two angles are not complementary, then their measures do not sum to 90.
Contrapositive	Exchange **AND** negate hypothesis and conclusion	$\sim q \rightarrow \sim p$	"if not q, then not p"	If the sum of two angles is not 90, then the angles are not complementary.

KEY CONCEPTS

Sometimes, the four statements do not have the same truth value, but the conditional and the contrapositive still do. Again, they are logically equivalent.

Statement	Symbols	Example
Conditional	$p \to q$	If a figure is a circle, then it is not a polygon. True
Converse	$q \to p$	If a figure is not a polygon, then it is a circle. False (oval)
Inverse	$\sim p \to \sim q$	If a figure is not a circle, then it is a polygon. False (oval)
Contrapositive	$\sim q \to \sim p$	If a figure is a polygon, then it is not a circle. True

CHALLENGE AND EXTEND

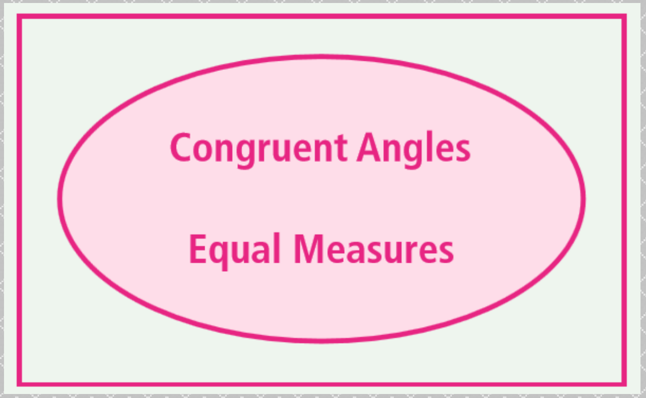
1. Write the sentence as a conditional.
2. Then write the converse, the inverse and the contrapositive for the conditional.
3. Assign a truth value to each statement.
4. Use the related statements to draw a Venn diagram that represents the relationship between congruent angles and their measures.

Two congruent angles have the same measure.

CHALLENGE AND EXTEND

Answer:

Conditional: If two angles are congruent, then they have the same measure. True
Converse: If two angles have the same measure, then they are congruent. True
Inverse: If two angles are not congruent, then they do not have the same measure. True
Contrapostive: If two angles do not have the same measure, then they are not congruent. True



SUMMARY

- A conditional is an *if-then* statement
- The symbolic form of conditional is $p \rightarrow q$ where p is the hypothesis and q is the conclusion.
- A given conditional can be rewritten in the following forms: converse, inverse, contrapositive.
- Each of the statements has a truth value.
- Related statements with the same truth value are logically equivalent.
- The conditional and contrapositive are logically equivalent.

LEARNING RUBRIC

- Got It: Applies truth values to if-then statements and identifies logical equivalence
- Almost There: Write the converse, inverse, and contrapositive of a given conditional
- Moving Forward: Write conditionals from given statements or venn diagrams
- Getting Started: Identifies the hypothesis and conclusion of a given conditional or venn diagram

HOMEWORK

- 2-1 Pages 77-79: 12, 16, 22, 24, 26, 38
- 2-2 Pages 85-86: 14-22 even; 32, 36, 42